Admirals and Raleigh Calculation policy, April 2024

Aims:

The purpose of this calculation policy is to outline the principles and strategies for teaching calculation in line with the Mastery Mathematics approach developed by the National Centre for Excellence in the Teaching of Mathematics (NCETM). This policy aims to provide students with a deep understanding of mathematical concepts and develop their ability to apply calculation techniques accurately and efficiently. We use NCETM, curriculum prioritisation materials from Nursery all the way up to Year 6 to enable children to leave us with a secure mathematical understanding. By adopting this policy, we aim to ensure students gain a deep understanding of mathematical concepts, develop procedural fluency, and become confident problem solvers.

We apply the following principles to Maths Teaching and Learning at Raleigh and Admirals Academies:

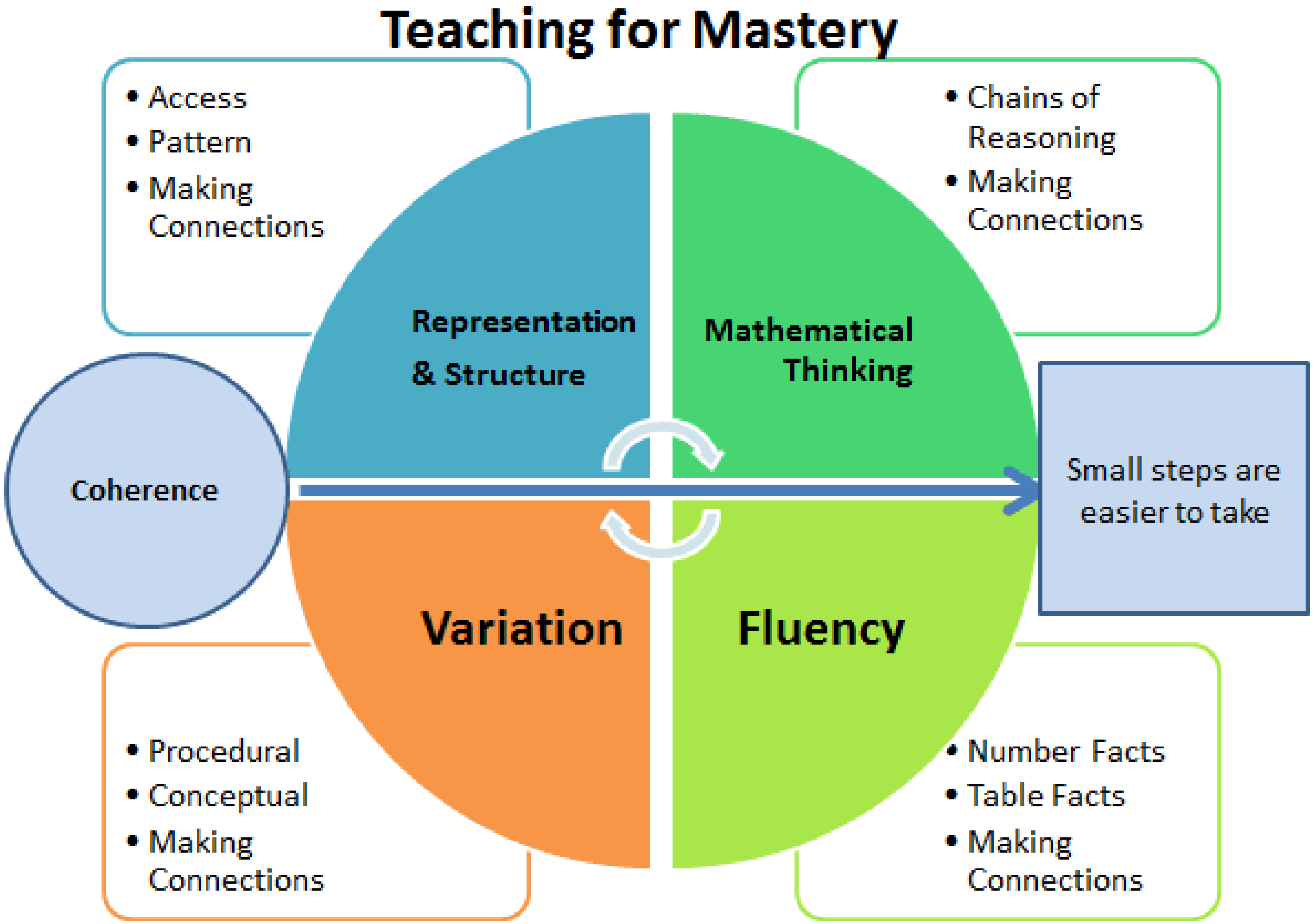
Mathematical Thinking: Emphasise the importance of understanding the underlying principles and concepts of each calculation method, rather than relying solely on memorisation.

Representation and Structure: Ensure a step-by-step progression of learning, where students build upon previously acquired knowledge and skills. Stem sentences describe the representation and helps our students move to working in the abstract.

Variation: Provide students with a range of problem-solving scenarios and contexts to develop their flexibility in applying calculation techniques.

Fluency: Promote the development of procedural fluency alongside the ability to reason mathematically and solve real-world problems.

Concrete, Pictorial, Abstract (CPA): Use a variety of representations, including concrete manipulatives, pictorial diagrams, and abstract symbols, to facilitate students' understanding and transition between different stages of learning.



How to use this policy:

- Use the policy as the basis of your planning but ensure you use previous or following stages' guidance to allow for personalised learning- this can be found using the progressions of skills documents in this policy.
- Cross reference with the National Curriculum end of year number skills expectations for each year group
- Use Assessment for Learning to identify suitable next steps in calculation for groups of children.
- If, at any time, children are making significant errors, return to the previous step to make sure that learning gaps are not created.
- Always model a CPA approach to allow children to see the links and to make rich connections.
- Teach addition and subtraction at the same time to demonstrate how they link and the inverse law.
- Teach multiplication and division at the same time to demonstrate the links between these operations and the inverse law.
- All written methods should be presented to the children alongside resources and images to ensure that children develop their conceptual understanding of the written method being taught.

Addition and Subtraction:

Concrete Stage: Begin with the use of concrete manipulatives (e.g., base ten blocks) to represent and physically manipulate quantities.

Pictorial Stage: Progress to pictorial representations (e.g., number lines, bar models) to support students in visualising and understanding the calculation process.

Abstract Stage: Introduce formal written methods (e.g., column addition and subtraction) when students have a secure understanding of the underlying concepts and strategies.

Progressions of Skills in Addition and Subtraction

Number Bonds:

Year 1	Year 2
represent and use number bonds and related subtraction facts within 20	recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

Mastering Number Programme (KS1)

We have adopted this programme in our EYFS and KS1 classrooms to support the development of fluency at a young age. We want to foster the automaticity that will form the foundation of the building blocks we add as they enter KS2. The aims of this programme are listed in the table below:

For pupils	For teachers
• develop **fluency** in calculation and a **confidence and flexibility** with number that exemplifies **good number sense** • be able to clearly **communicate** their mathematical ideas • demonstrate a **positive attitude** towards maths	• develop a secure understanding of how to **build firm mathematical foundations** • practise strategies shared in central training to support children's progress • develop understanding of appropriate **manipulatives** to support teaching of mathematical structures

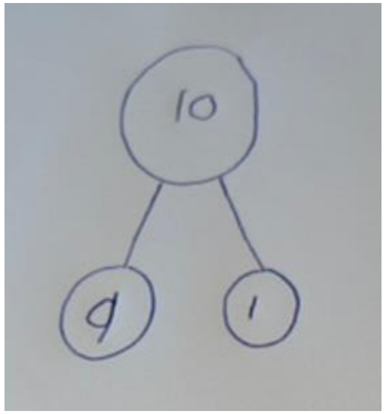
WRITTEN METHODS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Mental Calculation)		add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	

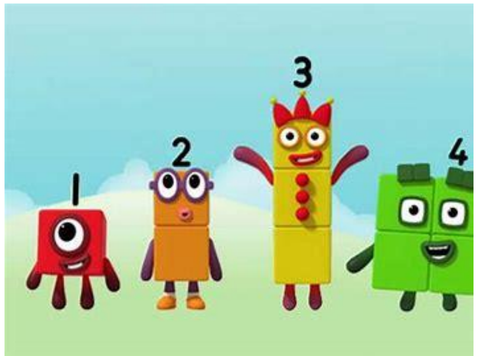
MENTAL CALCULATION					
add and subtract one-digit and two-digit numbers to 20, including zero	add and subtract numbers using concrete objects, pictorial representations, and mentally, including: * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers	add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds		add and subtract numbers mentally with increasingly large numbers	perform mental calculations, including with mixed operations and large numbers
read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs (appears also in Written Methods)	show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot				use their knowledge of the order of operations to carry out calculations involving the four operations

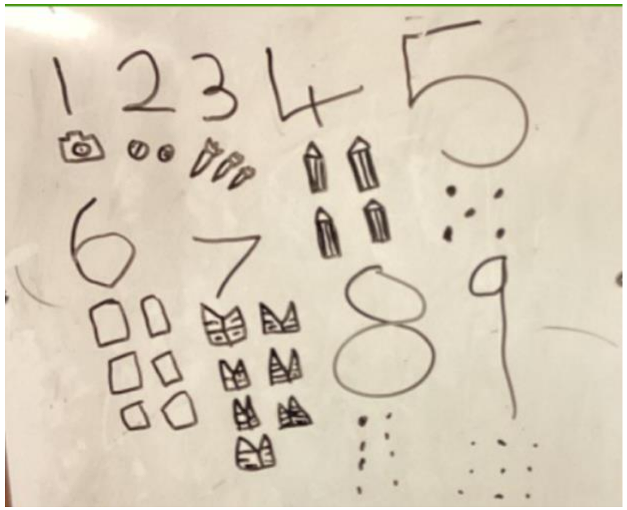
What Does this look like in our classrooms?

Please use the next few pages to see what the stages of learning within Addition and Subtraction look like for each year group.

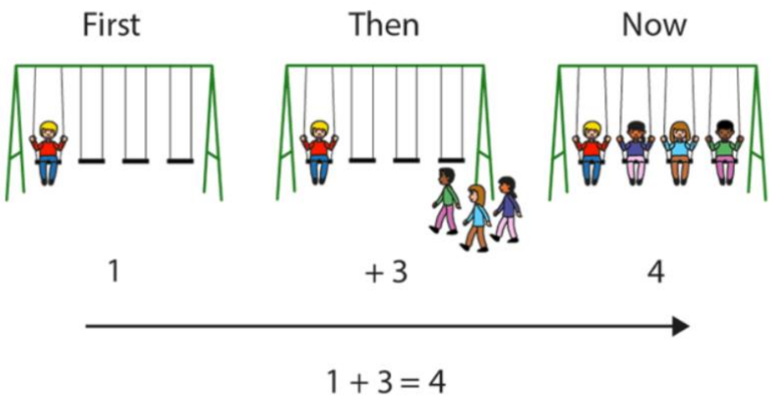
Nursery and Reception

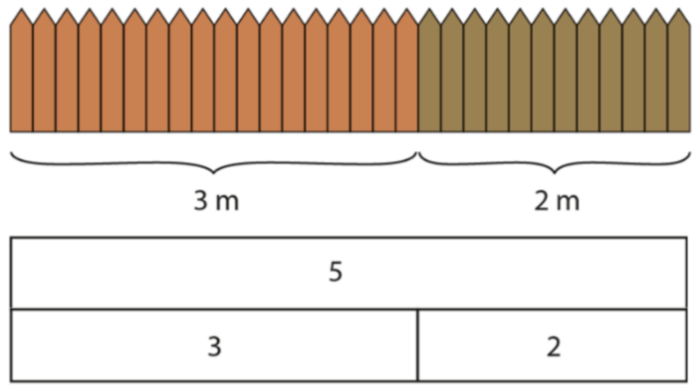


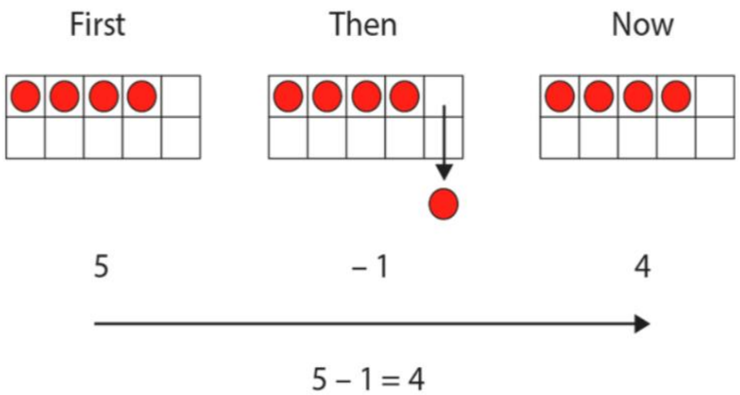




Year 1

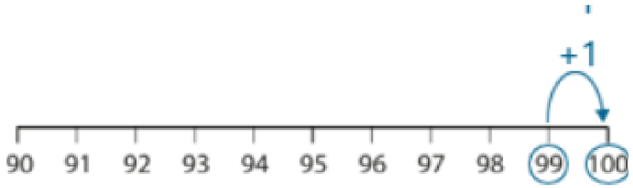


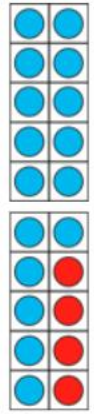




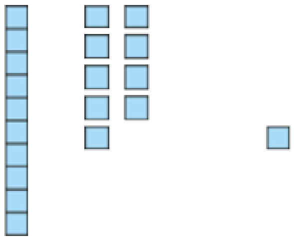
$$3 - 3 = 0$$

Year 2

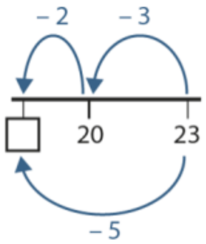




16 + 4 = 20



24 + 3 = 27



19 + 1

14 + 10 = 24

24 + 10 = 34

34 + 10 = 44

44 + 10 = 54

54 + 10 = 64

64 + 10 = 74

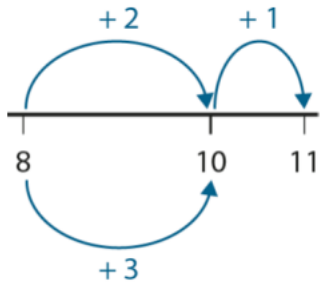
74 + 10 = 84

84 + 10 =

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

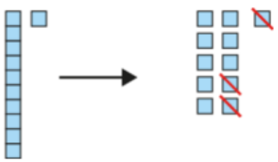
8 + 3

2 1



one less   one more

☐ ← 30 → ☐



11 – 3

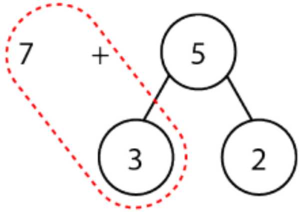
1 2

11 – 3 = 11 – 1 – 2

= 10 – 2

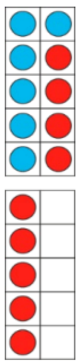
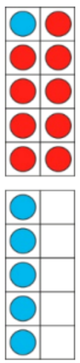
= 8

Year 3

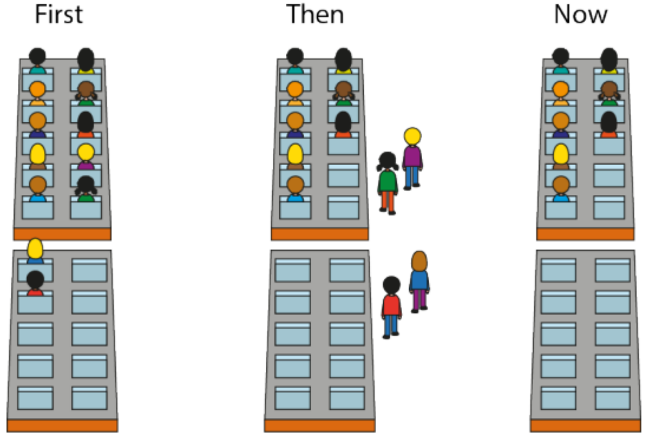


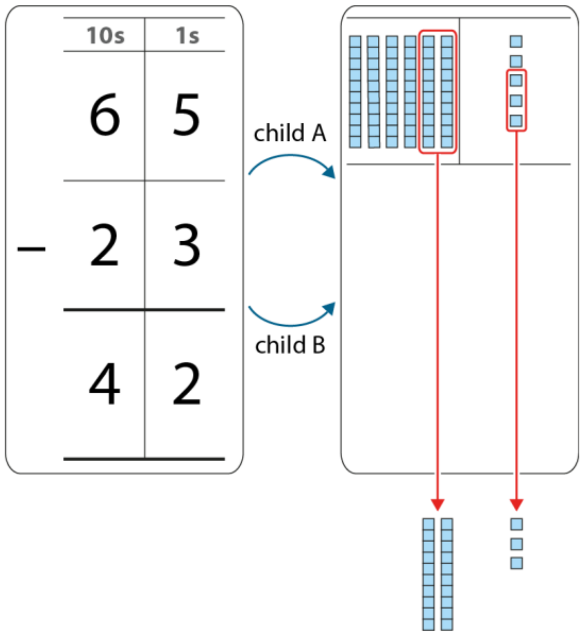
7 + 3 = 10

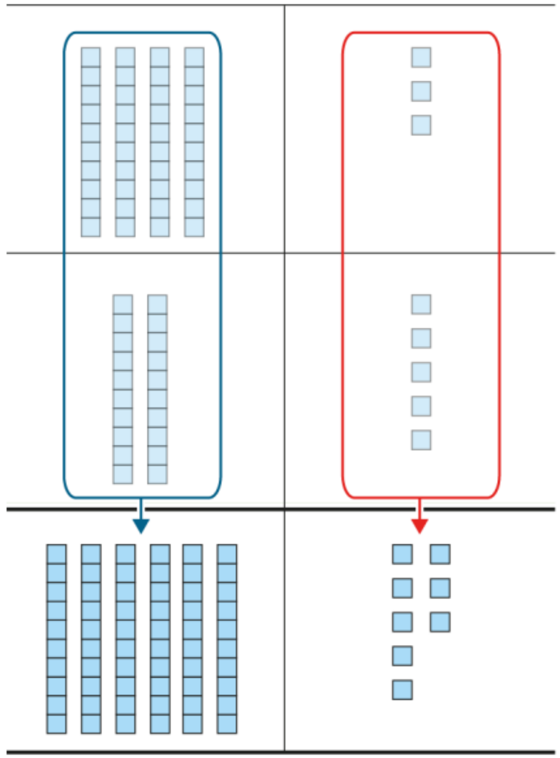
10 + 2 = 12

Subtraction *through* ten:	**Subtraction *from* ten:**
	
15 – 9 (9 partitioned into 5 and 4)	15 – 9 (15 partitioned into 10 and 5)
15 – 5 = 10 10 – 4 = 6	10 – 9 = 1 1 + 5 = 6

'First there were twelve children on the ride. Then four got off. Now there are eight children on the ride.'



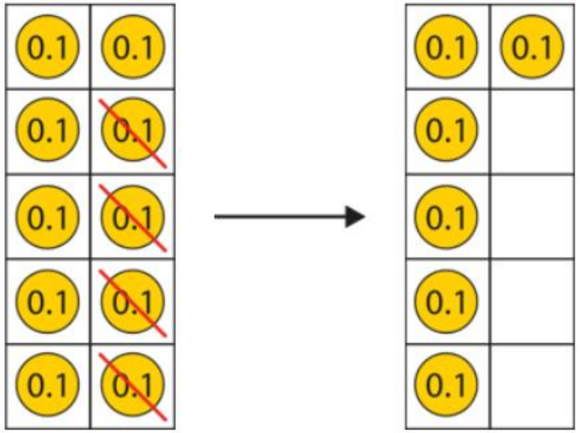




$$\begin{array}{r} 4\;3 \\ +\;2\;5 \\ \hline 6\;8 \end{array}$$

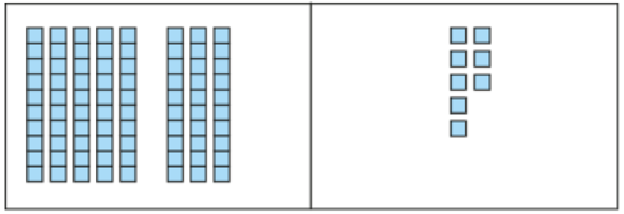
100s	10s	1s
~~2~~ 1	1 2	3
- 1	4	2

Year 4

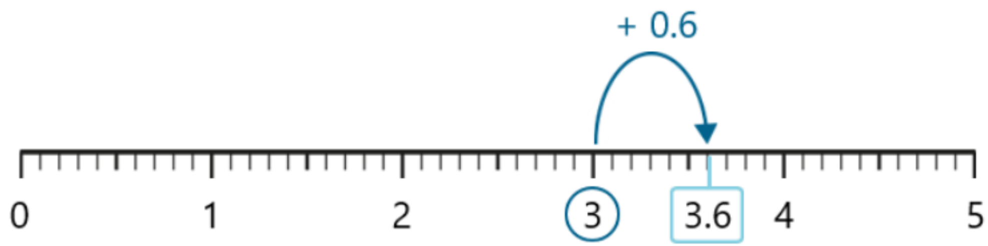


1.0 − 0.4 = 0.6

$$\begin{array}{rr} & 2\ 5 \\ + & 4\ 7 \\ \hline & 7\ 2 \\ & {}_1\ \ \end{array}$$



	10s	1s
	~~9~~ 8	14
−		6
	8	8



6,584 + 2,739= 9,323

$$\begin{array}{rl} & 6,\ 5\ 8\ 4 \\ + & 2,\ 7\ 3\ 9 \\ \hline & 9,\ 3\ 2\ 3 \\ & {}_1\ \ {}_1\ \ {}_1 \end{array}$$

1,000 p = £10									
100 p	100 p	100 p	100 p	100 p	100 p	100 p	100 p	100 p	100 p

Year 5

£1.37 + £2.45 = £3.82

£3.82	
£1.37	£2.45

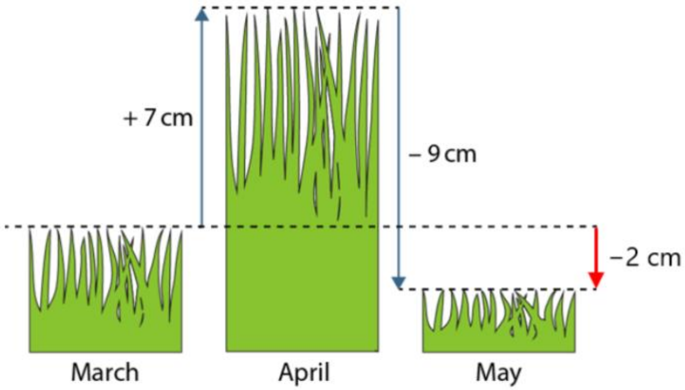
	£	1	.	3	7
+	£	2	.	4	5
	£	3	.	8	2
				1	

£11.05 − £3.50 = £7.55

		0	10		
	£	~~1~~	~~1~~	.	10 5
−	£		3	.	5 0
	£	0	7	.	5 5

	0	14		10		
	~~1~~	~~5~~	.	~~1~~	10	
−		8	.	2	8	
		6	.	8	2	

15.1 − 8.28 = 6.82

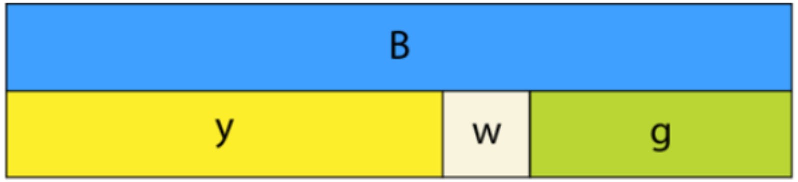


7 − 9 = −2

?		
30 minutes	20 minutes	55 minutes

30 + 20 + 55 = 50 + 55
= 105
60 45
= 1 hr 45 mins

Year 6



$B = y + w + g$

?		
30 minutes	20 minutes	55 minutes

643,801 + 505,370 = 1,149,171

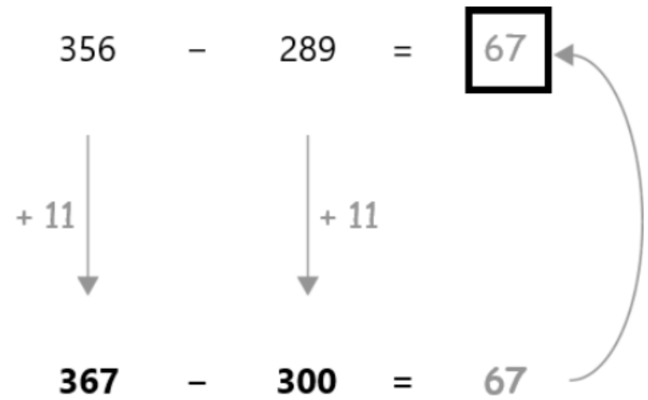
```
    6 4 3 , 8 0 1
+   5 0 5 , 3 7 0
  1 , 1 4 9 , 1 7 1
  1       1
```

357,022 − 243,610 = 113,412

```
  3 5 7⁶, ¹0 2 2
− 2 4 3 , 6 1 0
  1 1 3 , 4 1 2
```

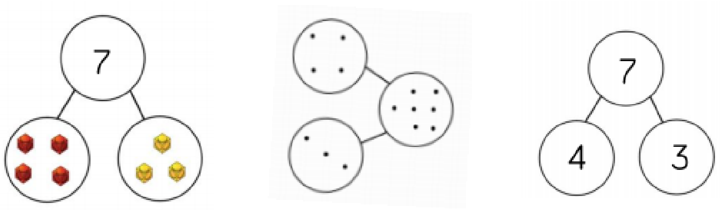
200,000 − 158,436 = 41,564

```
  2 0 0 , 0 0 0    − 1 →      1 9 9 , 9 9 9
− 1 5 8 , 4 3 6    − 1 →    − 1 5 8 , 4 3 5
                              0 4 1 , 5 6 4
```



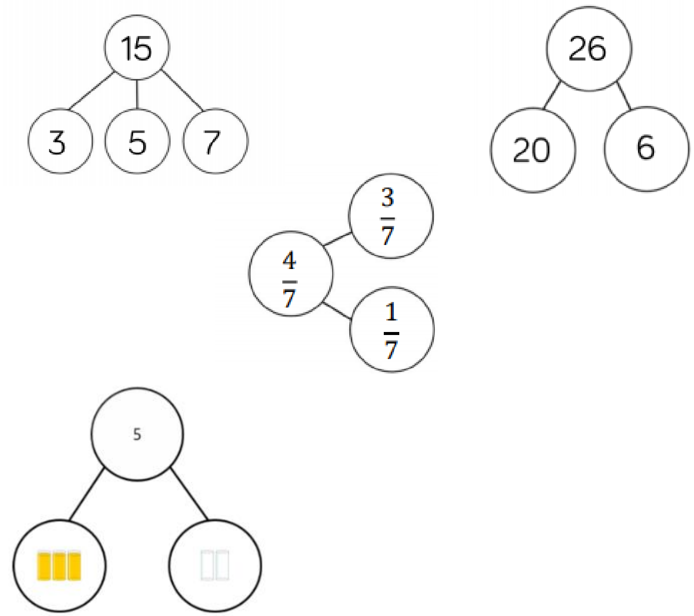
Overview of the different models – addition and subtraction

Part-Whole Model



$7 = 4 + 3$ $\quad$ $7 - 3 = 4$

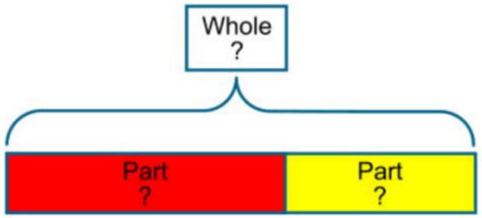
$7 = 3 + 4$ $\quad$ $7 - 4 = 3$



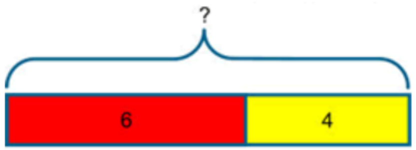
Key points

- Supports children in their understanding of partitioning numbers.
- A ‘whole’ can be represented by one object; if some of the whole object is missing, it is not the ‘whole’.
- A whole object can be split into two or more parts in many different ways. The parts might look different; each part will be smaller than the whole, and the parts can be combined to make the whole.
- A ‘whole’ can be represented by a group of discrete objects. If some of the objects in the group are missing, it is not the whole group – it is part of the whole group.
- A whole group of objects can be composed of two or more parts, and this can be represented using a part–part–whole ‘cherry’ diagram. The group can be split in many different ways. The parts might look different; each part will be smaller than the whole group and the parts can be combined to make the whole group.

Bar Models

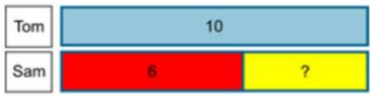


Addition



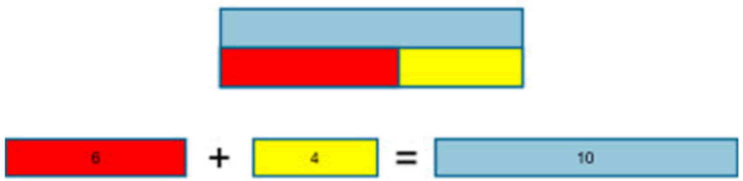
I have 6 red pencils and 4 yellow pencils. How many pencils do I have?

Subtraction



Tom has 10 pencils and Sam has 6 pencils. How many more does Tom have?

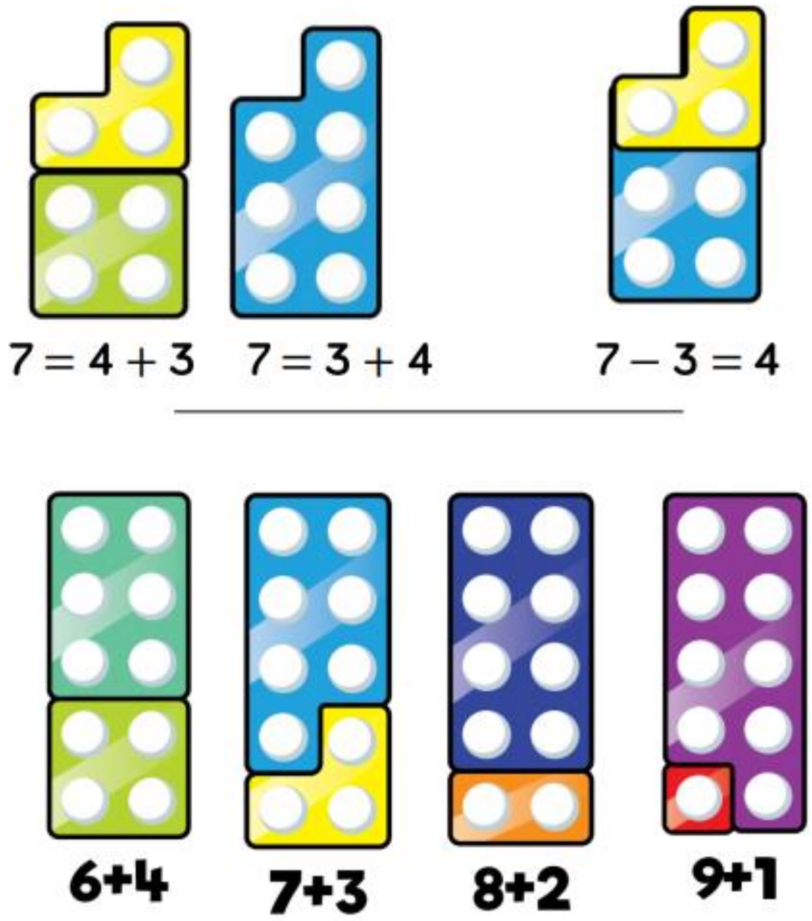
Equivalence



Key points

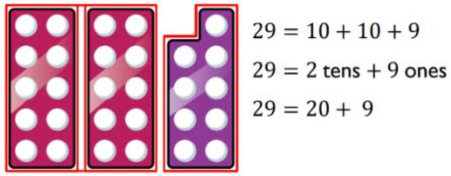
- Cuisenaire rods, cubes and counters and can be used in a line as a concrete representation of the bar model.
- The bar model is used in teaching for mastery to help children to 'see' mathematical structure. It is not a method for solving problems, but a way of revealing the mathematical structure within a problem and gaining insight and clarity to help solve it. It supports the transformation of real-life problems into a mathematical form and can bridge the gap between concrete mathematical experiences and abstract representations.
- Pupils need to develop fluency in using this structure to represent addition and subtraction problems in a variety of contexts using the bar model.
- The model will help children to see that different problems share the same mathematical structure and can be visualised in the same way. Asking children to write their own problems, using the bar as the structure will help to consolidate this understanding.

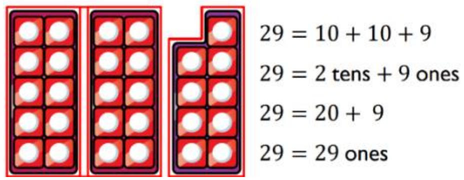
Numicon



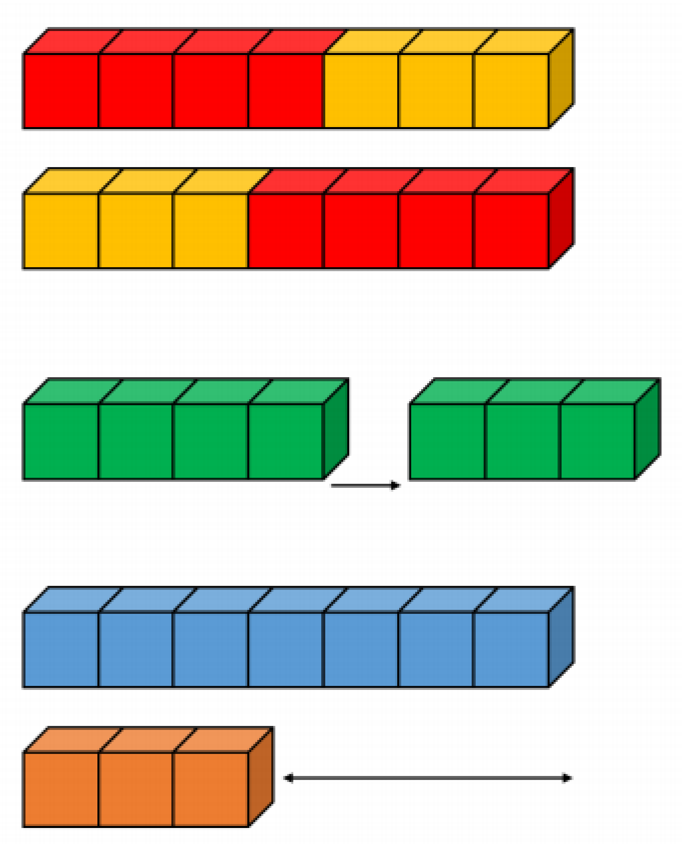
Key points

- A great resource to help children subitise numbers as well as explore aggregation, partitioning and number bonds.
- When adding, children can see how the parts come together to make a whole.
- When subtracting, children can start with the whole and then place one of the parts on top of the whole to see what part is missing.
- Numicon is used to expose the pairs and evenness in multiples of two. The non-example (five) is used to expose how this is different from odd numbers.
- The ten Numicon is used greyed out to expose the '10 and' nature of the teen numbers.





Cubes



$7 = 4 + 3$

$7 = 3 + 4$

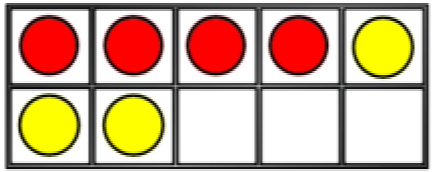
$7 - 3 = 4$

$7 - 3 = 4$

Key points

- Useful when adding and subtracting one-digit numbers.
- When adding, children can see how the parts come together to make a whole.
- Children can use two different colours to represent the two parts before joining them to create the whole.
- When subtracting, children can start with the whole and then remove the number of cubes that they are subtracting to find the answer. This model of subtraction is reduction or take away.
- To find the difference, both numbers can be made up and then lined next to each other.
- Useful when working with smaller numbers.

Ten Frames (within 10)



$4 + 3 = 7$

$3 + 4 = 7$

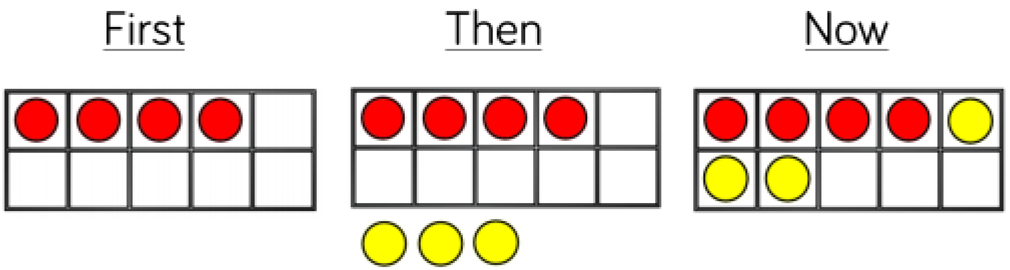
$7 - 3 = 4$

$7 - 4 = 3$

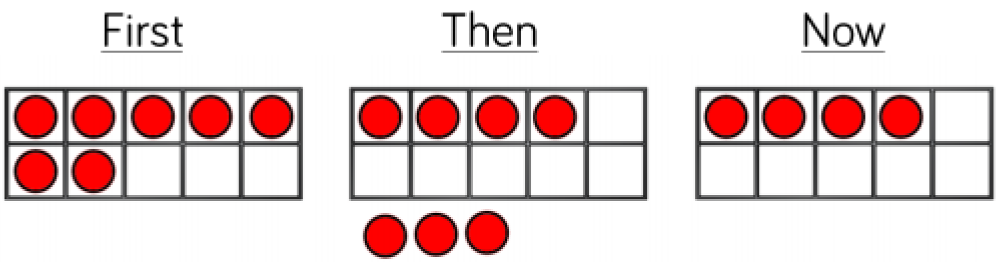
4 is a part.

3 is a part.

7 is the whole.



$4 + 3 = 7$

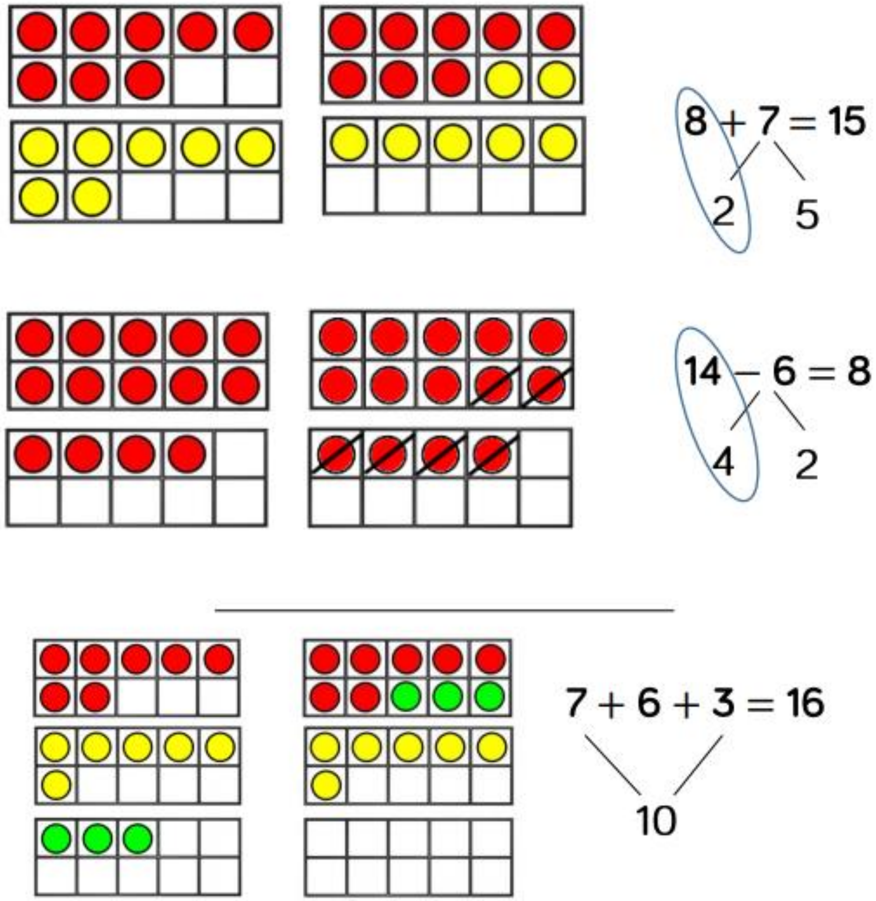


$7 - 3 = 4$

Key points

- When adding and subtracting, the ten frame can support children to understand the different structures of addition and subtraction.
- Linking parts and wholes to the items on the ten frame introduces children to aggregation and partitioning.
- Subtraction can be represented on a tens frame. As shown by the first, then and now stages.
- Adding a story structure can help children understand the change.
- First there were 7 apples. Then, 3 apples were eaten. Now, there are 4 apples left.

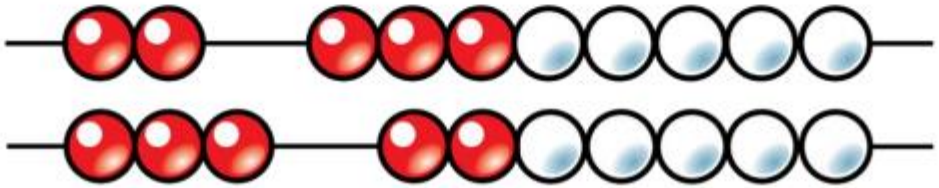
Ten Frames (within 20)

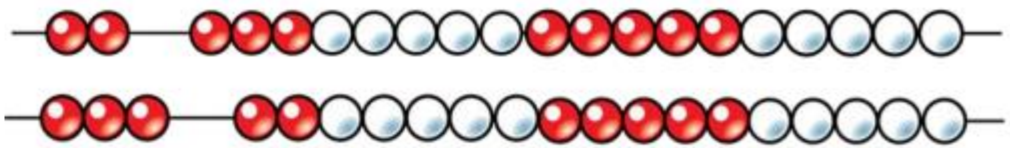


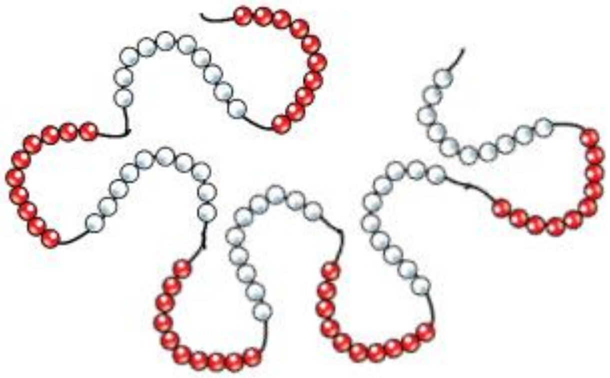
Key points

- When adding two single digits, children can make each number on a separate tens frame before moving part of one number to make 10.
- This shows children how they have partitioned one of the numbers to make 10, which makes links to effective mental methods of addition.
- When subtracting, firstly make the larger number on 2 ten frames. Then, remove the smaller number and think how you have partitioned the number to make 10. This supports mental methods of subtraction.
- When adding three single-digit numbers, children can make each number on a separate ten frame.
- Then, they can look to see if they can make a number bond to 10, which would make the calculation easier.
- Here, the ten frames support mental methods of addition and commutativity.

Bead Strings







Key points

10 bead strings

- Effective at helping children investigate number bonds to 10.
- Moving one bead at a time allows children to systematically find all the number bonds to 10, whilst also linking to partitioning. 2 + 8 = 10. 3 + 7 = 10.

20 bead strings

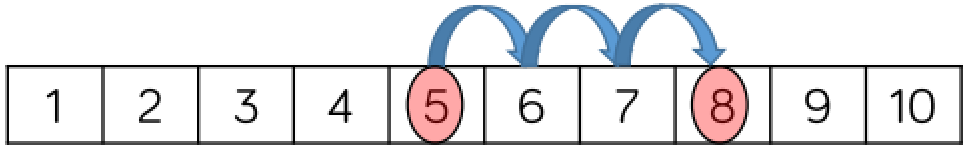
- Group beads into fives.
- Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20

100 bead strings

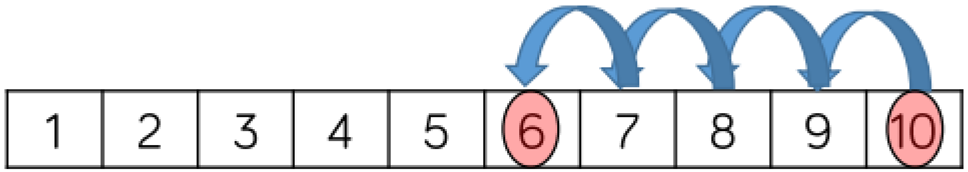
- Grouped in tens.
- Support number bonds to 100.
- Offer support when adding by making 10.
- Provide a link to adding to the next ten on number lines, which supports a mental method of addition.

Number Tracks

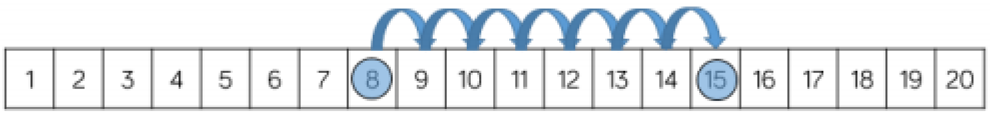
$5 + 3 = 8$



$10 - 4 = 6$



$8 + 7 = 15$

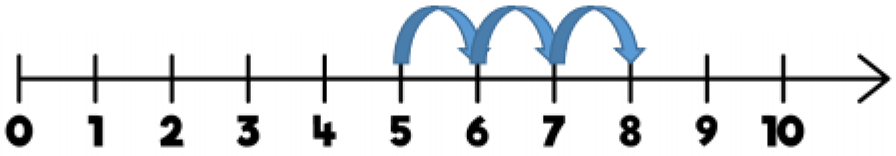


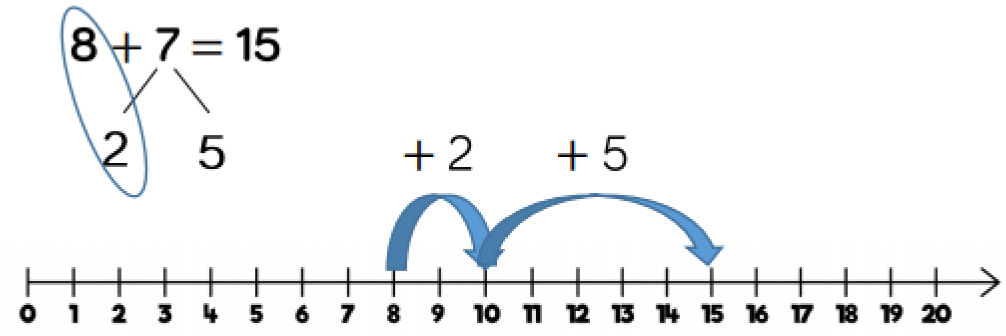
Key points

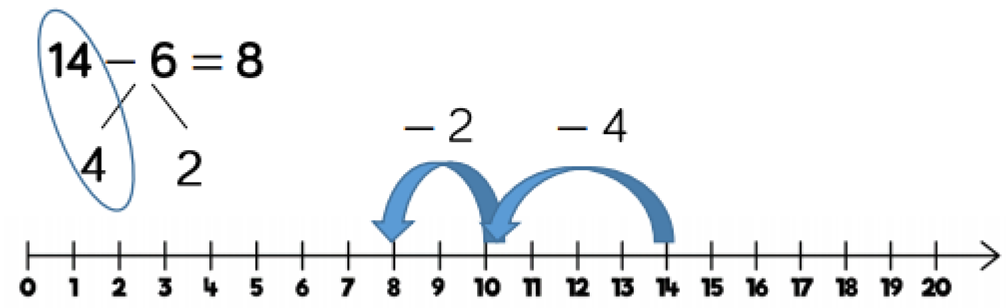
- Supports children's understanding of augmentation and reduction.
- When adding, children count on to find the total.
- Children can place a counter on the starting number and then count on to find the total.
- When subtracting, children count back to find their answer.
- They start at the minuend and then take away the subtrahend to find the difference.
- Work well alongside ten frames and bead strings as they all model counting on or counting back.

Number Lines (labelled)

$5 + 3 = 8$





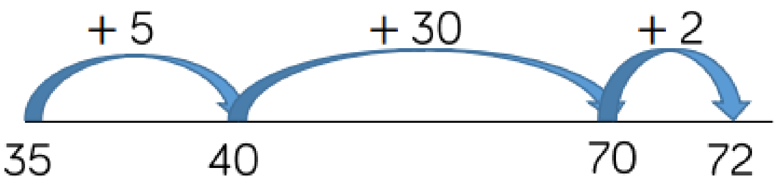


Key points

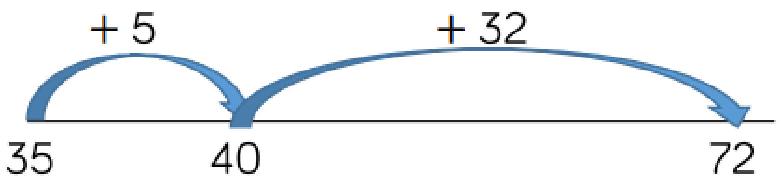
- Support children in their understanding of addition and subtraction.
- Start by counting on or back in ones (up or down the number line).
- Links directly to the use of a number track.
- Children can add numbers by jumping to the nearest 10 and then jumping to the total.
- This links to the making 10 method (also supported by ten frames).
- The smaller number is partitioned to support children when making a number bond to 10.
- Then, the remaining part is added.
- Children can subtract numbers by jumping to the nearest 10 first (supported by a ten frame).
- This shows children how they partition the smaller number into separate jumps.

Number Lines (blank)

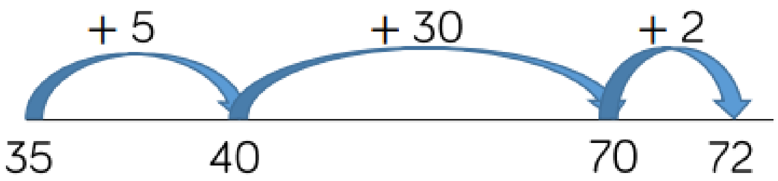
$35 + 37 = 72$



$35 + 37 = 72$



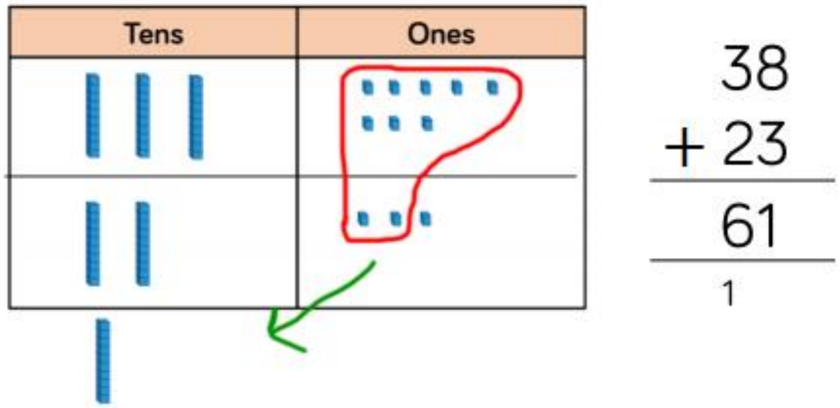
$72 - 35 = 37$

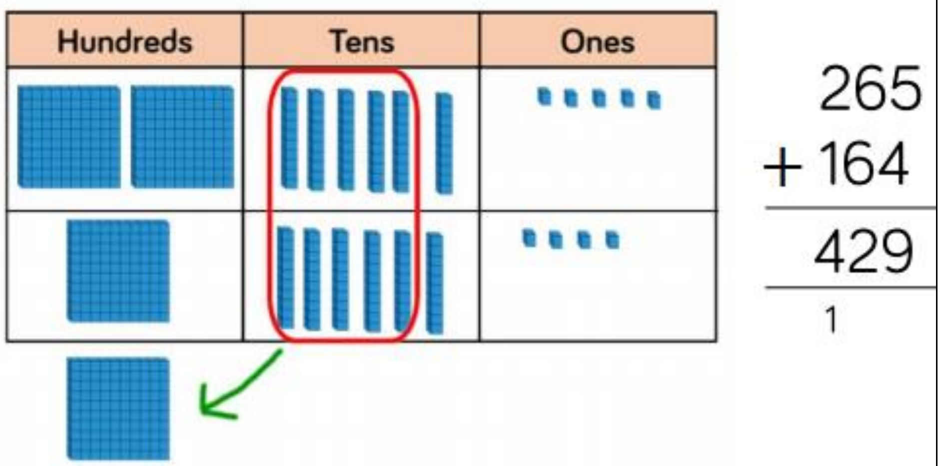


Key points

- Provide a structure for children to add and subtract numbers in smaller parts.
- Children can add by jumping to the nearest 10 and then adding the rest of the number as a whole or by adding tens and ones separately.
- Children can also count back on a number line to subtract. Firstly, they jump backwards to the nearest 10 and then subtracting the rest of the number.
- Blank number lines provide an effective way of finding the difference between numbers by counting on.
- Children start at the smallest number and count on to the largest number. They then add the parts they have counted on to find the difference.

Base 10/Dienes (addition)

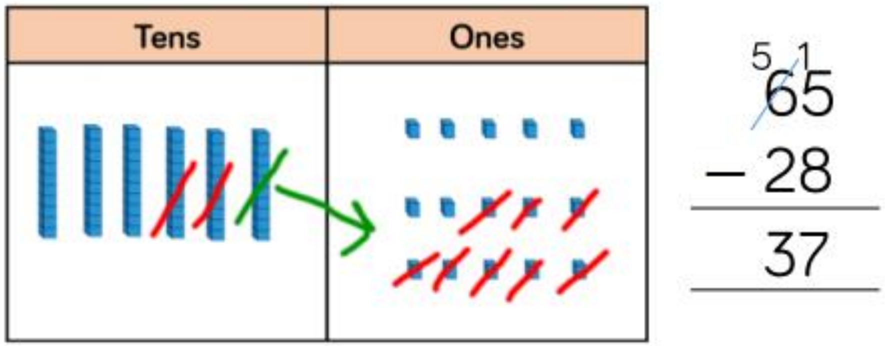


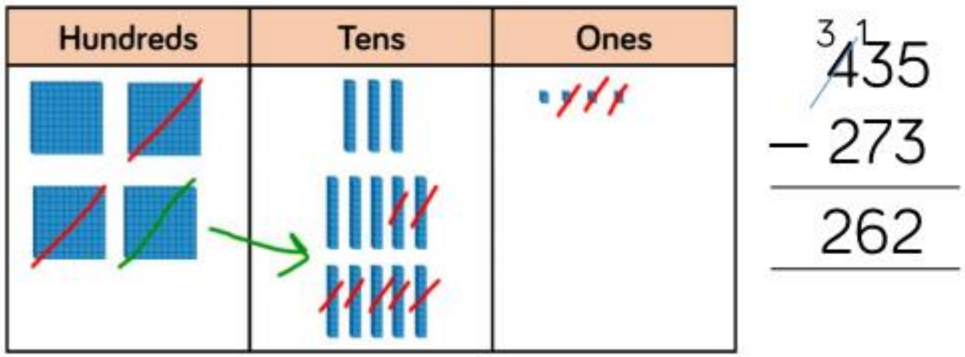


Key points

- Provides an effective way to support children’s understanding of column addition.
- It is imperative that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the model and the written method.
- Children should first add without exchange.
- This representation becomes less efficient with larger numbers.
- The next step is to use place value counters.
- When adding, always add from the smallest place value column.
- Key questions to ask the children:
- How many ones are there altogether?
- Can we make an exchange? (Yes or No).
- How many do we exchange? (10 ones for 1 ten)
- Show the exchanged 10 in the tens column by drawing and writing 1 below the column.
- How many ones do we have left? (Write the digit in the ones column).
- Repeat for each column

Base 10/Dienes (subtraction)

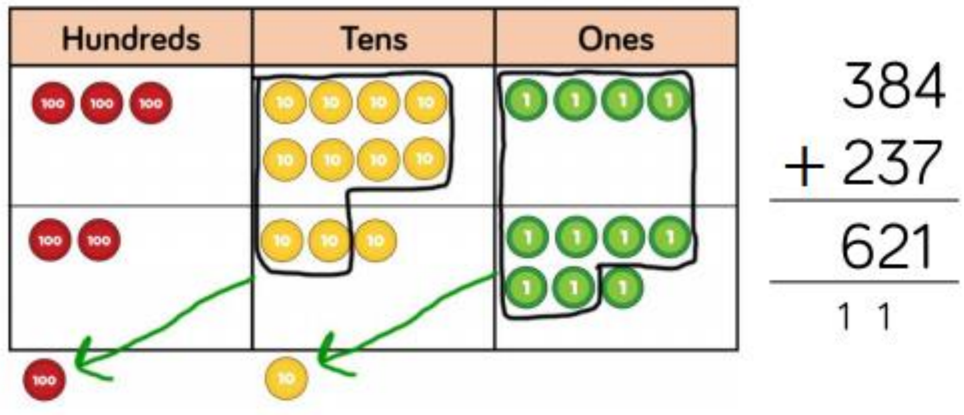


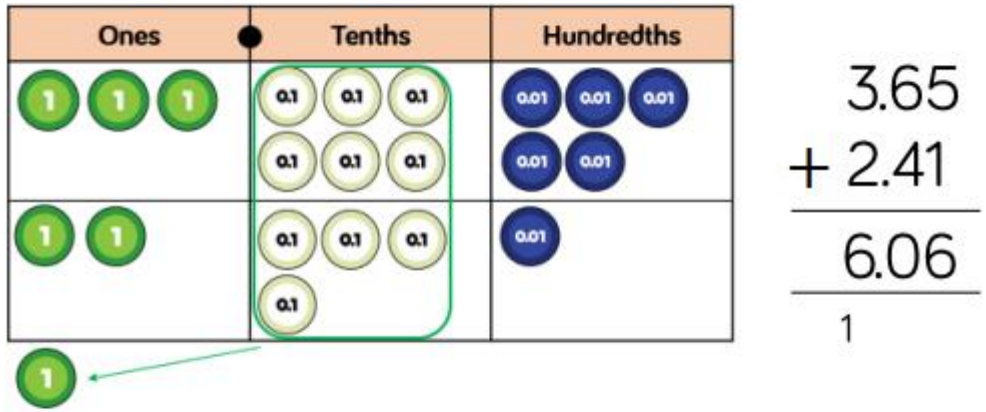


Key points

- Provides an effective way to support children's understanding of column subtraction.
- Children should first subtract without exchange.
- When building the model, children should just make the minuend (a number or quantity from which another number is subtracted) using Base 10.
- Then, they subtract the subtrahend (a number to be subtracted from another).
- Highlight this difference to addition to avoid errors by making both numbers.
- Children start by subtracting from the smallest place value column.
- When there are not enough ones/tens/hundreds to subtract in a column, children need to exchange from the column to the left (1 ten for 10 ones).
- They can then subtract efficiently.
- This model is efficient with up to 4-digit numbers.
- Place value counters are more efficient with larger numbers.

Place Value Counters (addition)

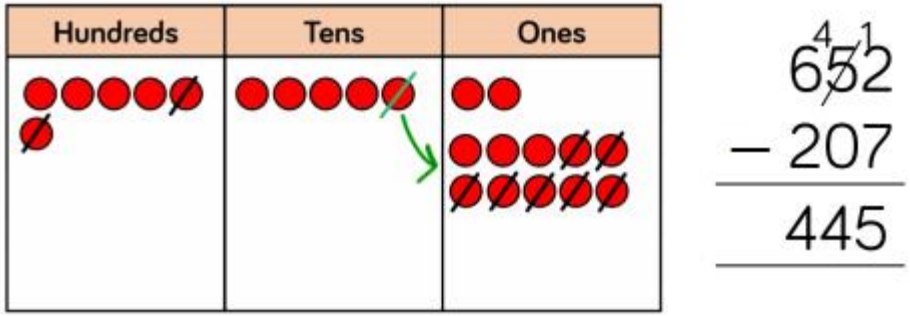


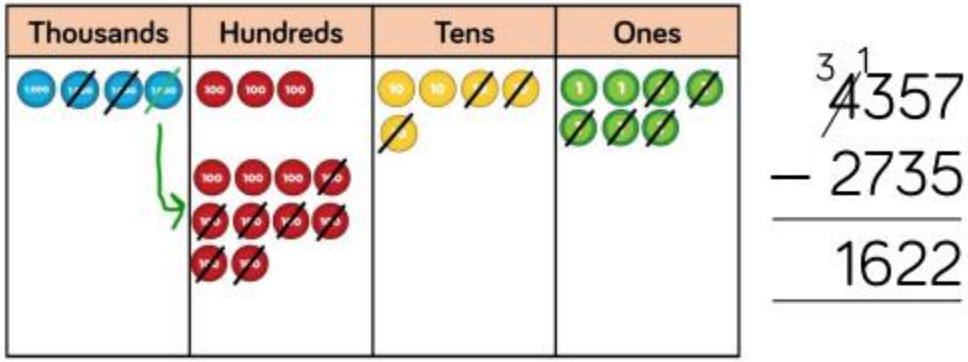


Key points

- Provides an effective way to support children's understanding of column addition.
- First add without exchange.
- Different place value counters can be used to represent larger numbers or decimals.
- Normal counters on a place value grid can also be used to enable children to experience the exchange between columns.
- When adding money, children can also use coins to support their understanding.
- It is important to show children the links between the coins when using the written method to support the addition of decimal amounts.

Place Value Counters (subtraction)

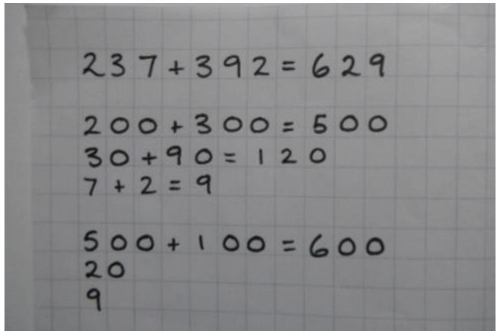


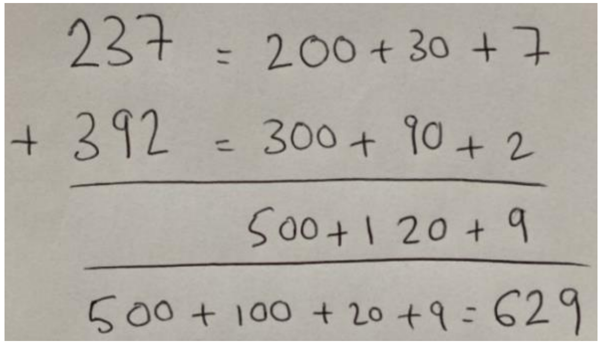


Key points

- Provides an effective way to support children's understanding of column subtraction.
- It is imperative that children write out their calculations alongside using or drawing counters to develop their conceptual understanding.
- First subtract without exchange.
- Normal counters on a place value grid can also be used to enable children to experience the exchange between columns.
- When building the model, children should just make the minuend (a number or quantity from which another number is subtracted) using counters.
- Then, they subtract the subtrahend (a number to be subtracted from another).
- Highlight this difference to addition to avoid errors by making both numbers.
- Children start by subtracting from the smallest place value column.
- When there are not enough ones/tens/hundreds to subtract in a column, children need to exchange from the column to the left (1 ten for 10 ones).
- They can then subtract efficiently.

Partitioning





Key points

- Partitioning is a way of splitting numbers into smaller parts to make them easier to work with.
- Partitioning links closely to place value.
- For example, the number 54 represents 5 tens and 4 ones.
- This shows how the number can
  be partitioned into 50 and 4.
- Partitioning can be used horizontally or vertically as show by the pictures

Multiplication and Division

Concrete Stage: Start with concrete materials (e.g., arrays, counters) to explore and model multiplication and division concepts.

Pictorial Stage: To use pictorial representations (e.g., arrays, bar models) to reinforce understanding and facilitate the transition from the concrete to the abstract stage.

Abstract Stage: Introduce formal written methods (e.g., long multiplication, short division) once students have a solid grasp of the underlying principles.

Progressions of Skills in Addition and Subtraction

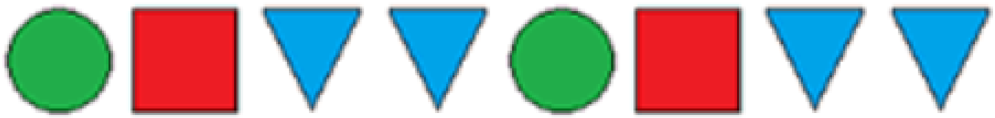
MULTIPLICATION & DIVISION FACTS					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
count in multiples of twos, fives and tens (copied from Number and Place Value)	*count in steps of 2, 3, and 5 from 0, and in tens from any number, forward or backward* (copied from Number and Place Value)	*count from 0 in multiples of 4, 8, 50 and 100* (copied from Number and Place Value)	*count in multiples of 6, 7, 9, 25 and 1 000* (copied from Number and Place Value)	*count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000* (copied from Number and Place Value)	
	recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers	recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	recall multiplication and division facts for multiplication tables up to 12 × 12		
MENTAL CALCULATION					
		write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Written Methods)	use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers	multiply and divide numbers mentally drawing upon known facts	perform mental calculations, including with mixed operations and large numbers
	show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot		recognise and use factor pairs and commutativity in mental calculations (appears also in Properties of Numbers)	multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	*associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^3/_8$)* (copied from Fractions)

WRITTEN CALCULATION					
Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
	calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs	write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods (appears also in Mental Methods)	multiply two-digit and three-digit numbers by a one-digit number using formal written layout	multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
				divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division where appropriate for the context divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
					use written division methods in cases where the answer has up to two decimal places (copied from Fractions (including decimals))

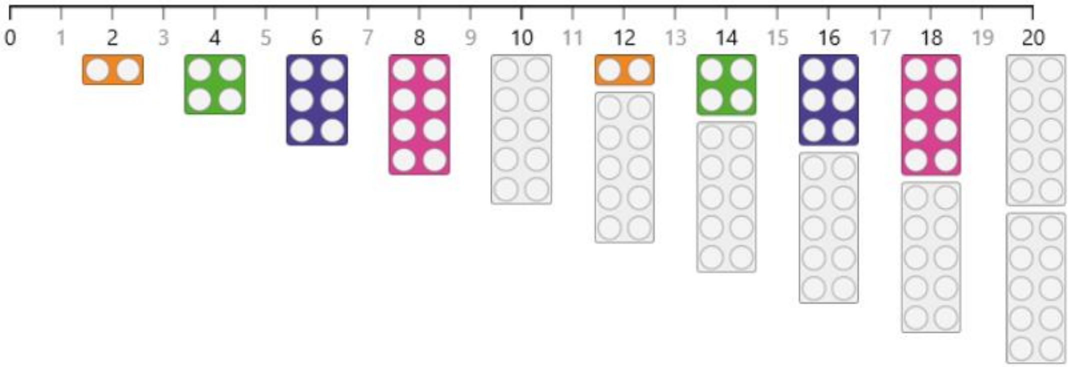
What Does this look like in our classrooms?

Please use the next few pages to see what the stages of learning within Multiplication and Division look like for each year group.

Nursery and Reception



<u>Year 1</u>

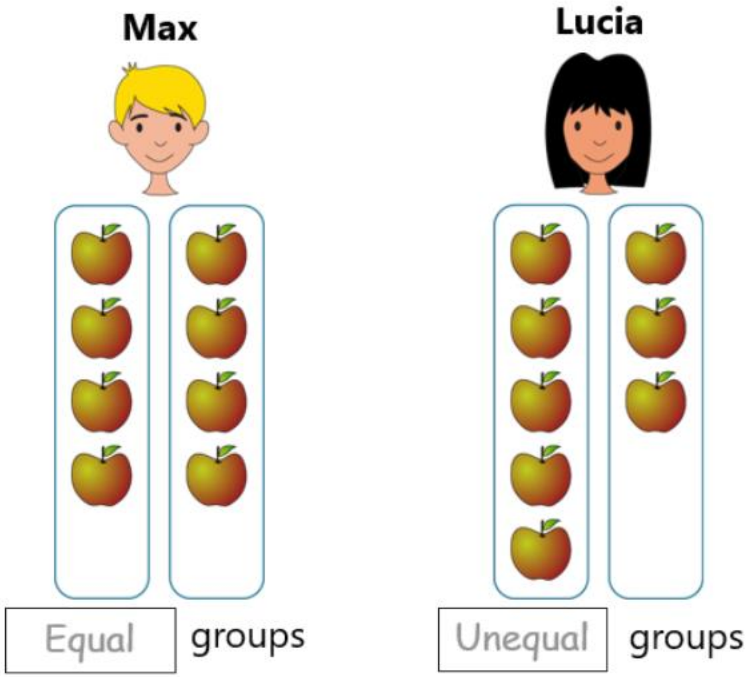


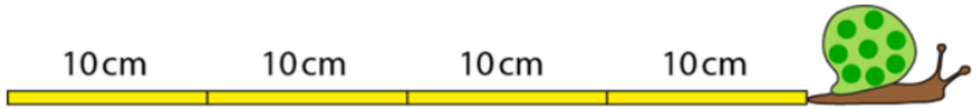
How many fingers (and thumbs) are there?
Count in groups of five.

How many wheels are there?
Count in groups of two.

nine ten-pennies

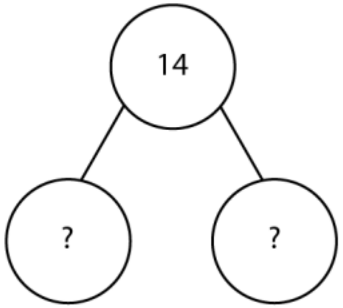
<u>Year 2</u>



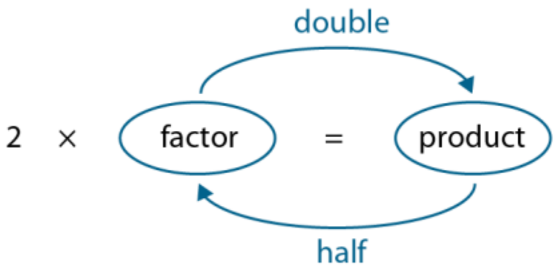


10 + 10 + 10 + 10

4 × 10

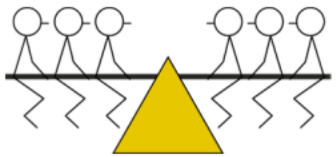


2 × 3



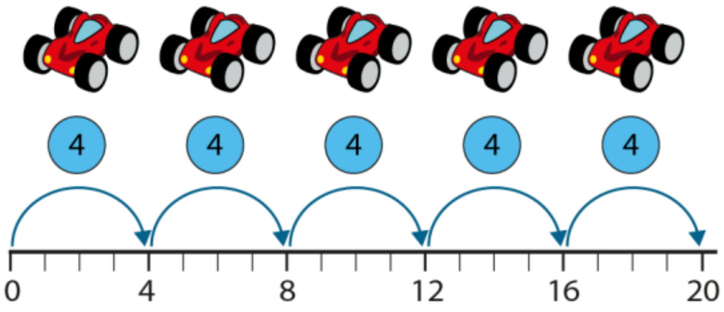
6 ÷ 2 = 3

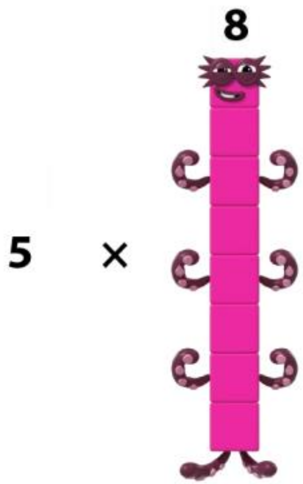
- *'Six divided between two is equal to three.'*
- *'Half of six is three.'*



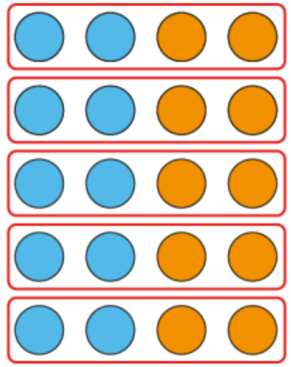
30	÷	5	=	6
dividend	÷	divisor	=	quotient

Year 3



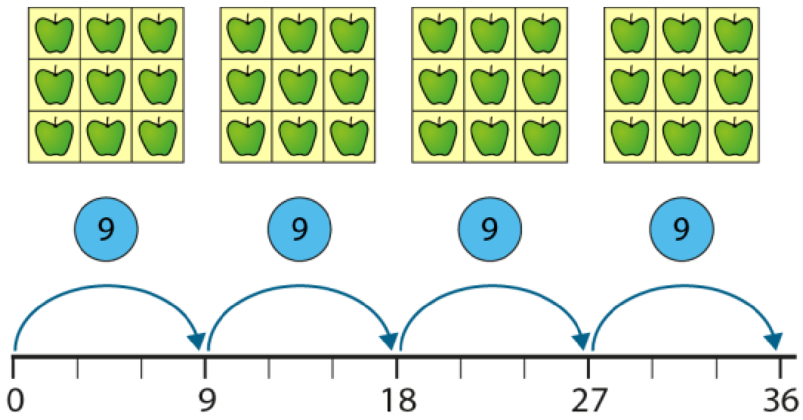


'How many fours are there?'



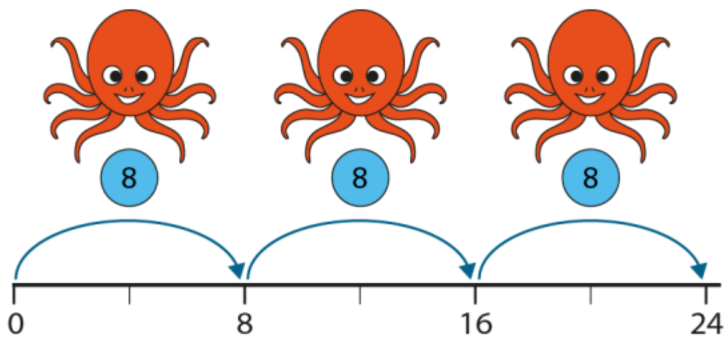
'There are five groups of four.'

'How many apples are there? Count in groups of nine.'



0	9	18	27	36							

'How many tentacles are there? Count in groups of eight.'



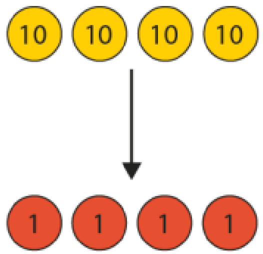
Year 4

$7 \times 13 = 7 \times 10 + 7 \times 3$

$= 70 + 21$

$= 91$

'I have forty. This is four tens. How much will I have if I divide by ten?'



$4 \text{ tens} \div 10 = 4 \text{ ones}$

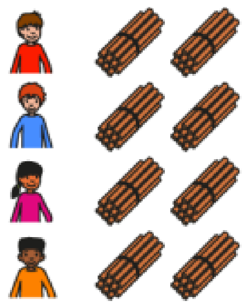
$40 \div 10 = 4$

1,000s	100s	10s	1s
			1
	1		

100 times the size

$\div 10$ →

	13

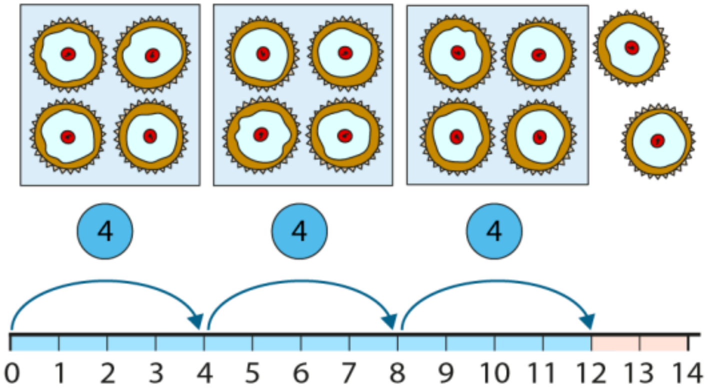


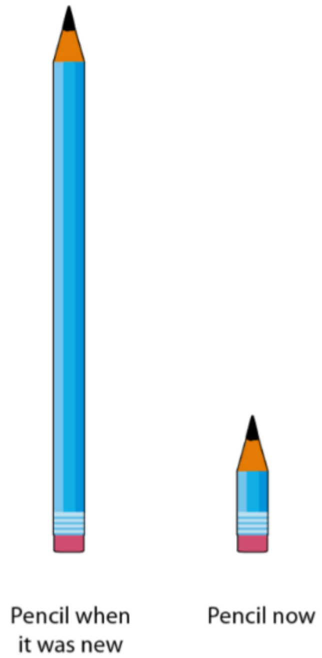
10s	1s
2	
4) 8	4

$8 \text{ tens} \div 4 = 2 \text{ tens}$

'Eight tens divided by four is equal to two tens.'

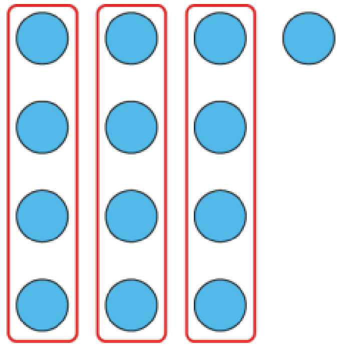
Year 5



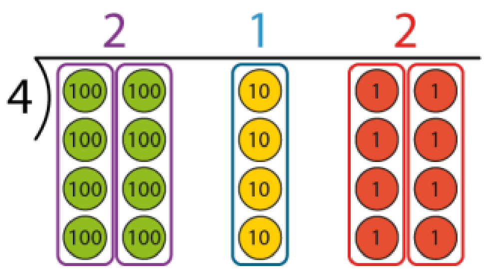


$20\text{ cm} \times \frac{1}{4} = 5\text{ cm}$

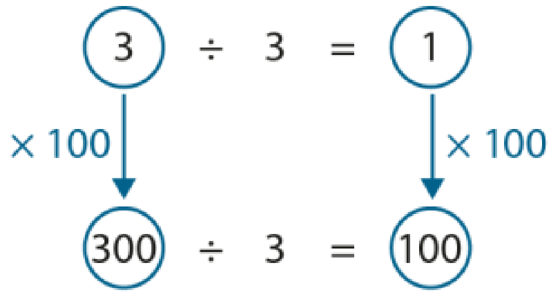
Dividend = 13, divisor = 4:

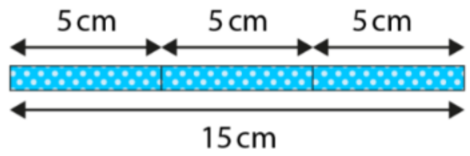


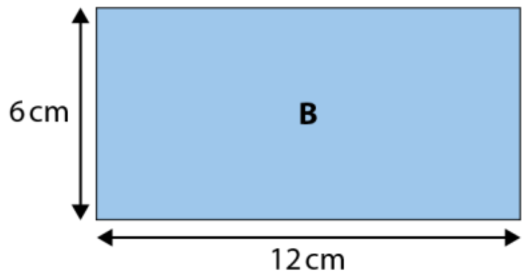
$13 \div 4 = 3 \text{ r } 1$



$$\begin{array}{r} 212 \\ 4\overline{)848} \end{array}$$







Year 6

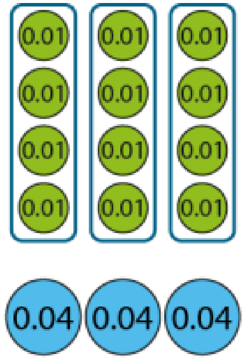
$6 \times 4 = 24$

6×4 ones $= 24$ ones

6×4 tenths $= 24$ tenths

$6 \times 0.4 = 2.4$

$$\begin{array}{r} 29\tfrac{5}{25} \\ 25\overline{)730} \\ \underline{50} \\ 230 \\ \underline{225} \\ 5 \end{array}$$

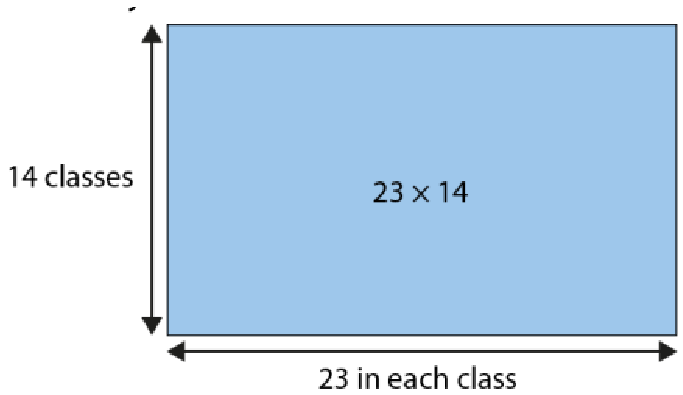


$3 \times 0.04 = 0.12$

$3 \times$ 4 hundredths $=$ 12 hundredths

$$\begin{array}{r} 972 \\ \times\quad 5 \\ \hline 4860 \\ \hline {\scriptstyle 3\,1} \end{array}$$

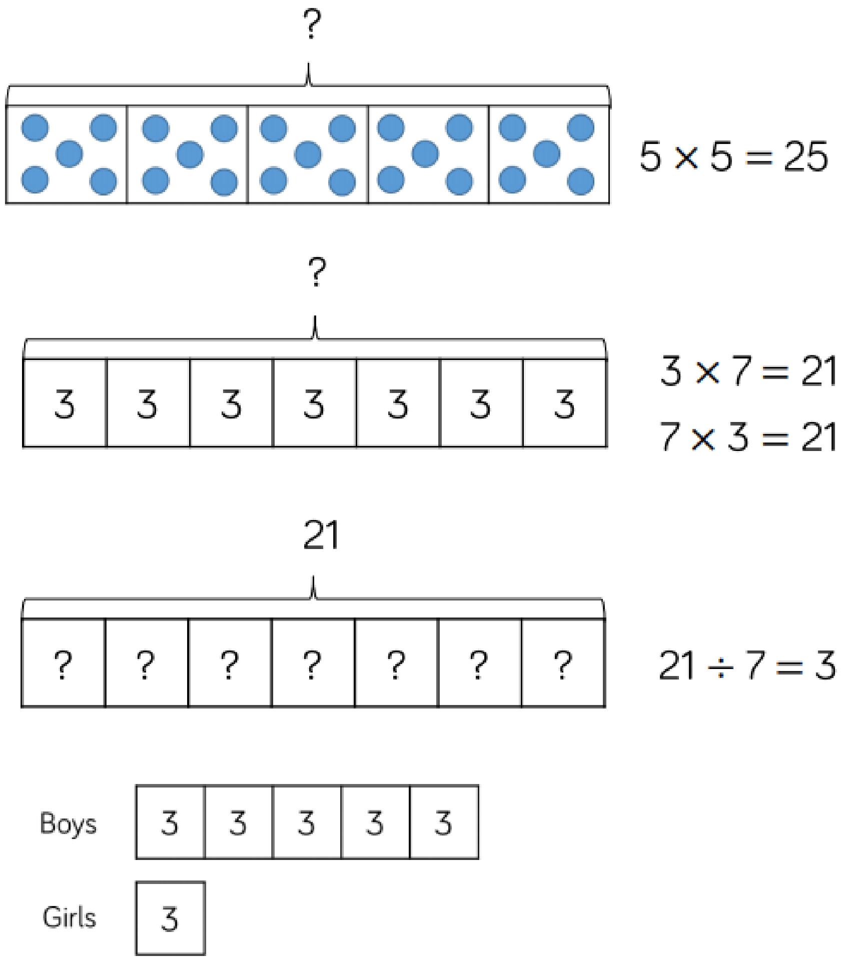
$$\begin{array}{r} 4.56 \\ \times\quad 4 \\ \hline 18.24 \\ \hline {\scriptstyle 2\;\;2} \end{array}$$



$$\begin{array}{r} 23.6 \\ 15\overline{)354.0} \\ \underline{30} \\ 54 \\ \underline{45} \\ 90 \\ \underline{90} \\ 0 \end{array}$$

Overview of the different models – multiplication and division

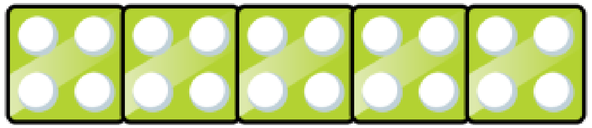
Bar Model



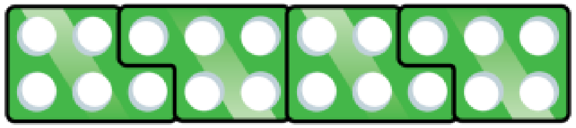
Key points

- The single bar model can be used to represent repeated addition.
- Counters, cubes, or dots can be placed/drawn within the bar model before moving onto digits.
- Division can be represented by showing the whole written above or in the top bar and then dividing the below bar into equal groups.
- Bar models are a great way for children to draw out and then represent what the knowns and unknowns are within a problem.
- Children then use an appropriate operation to solve the calculation.
- When working out scaling problems, more than one bar model is useful to represent this type of problem.
- For example, there are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?
- Multiple bar models provide an opportunity to compare the groups.

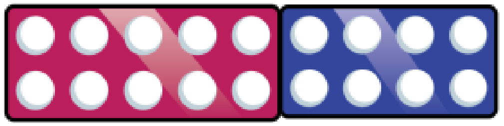
Numicon



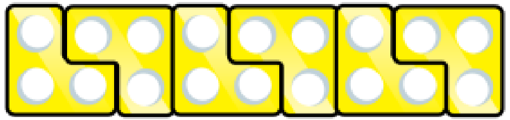
$5 \times 4 = 20$
$4 \times 5 = 20$



$5 \times 4 = 20$
$4 \times 5 = 20$



$18 \div 3 = 6$



Key points

- Numicon can be used to represent repeated addition.
- Multiplications can be built in a row by placing down the numicon.
- Can should interlock the shapes when using odd numbers.
- They can check the total by placing other numicon pieces on top, such as the tens pieces.
- The following patterns within multiplication can be seen when using numicon:
- Odd x odd = even
- Odd x even = odd
- Even x even = even
- Numicon can be used to support children's understanding of grouping when dividing.
- Children can make the number they are dividing and then place the number they are diving by over the top to find how many groups of the number there are.
- For example, there are 6 groups of 3 in 18.

<table>
<tr>
<td>

Bead Strings

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$

$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 5 = 3$

$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

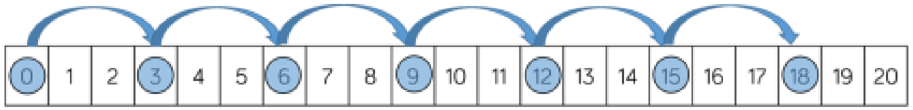
</td>
<td>

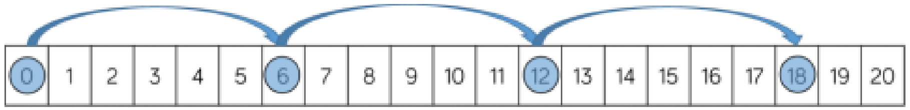
Key points

- Can support children in their understanding of multiplication as repeated addition.
- Encourage children to count in multiples as they build the number.
- Children can use the bead string to count forwards and backwards in multiples as they move the beads.
- Children can build the number they are dividing and then group the beads into the number they are dividing by.
- For example, 20 divided by 4. The children make 20 first. Then they group the beads into groups of 4. Finally, they count how many groups they have made to find the answer.

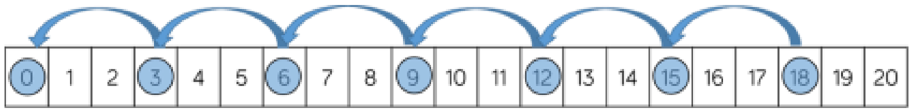
</td>
</tr>
</table>

Number tracks





$$6 \times 3 = 18$$
$$3 \times 6 = 18$$

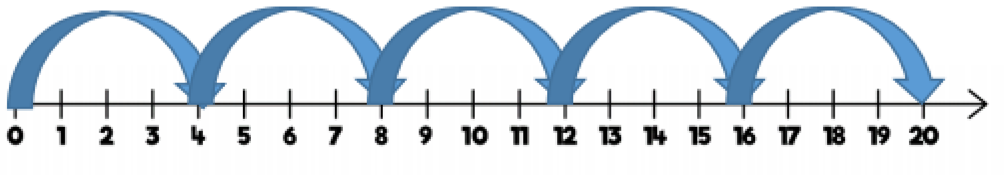


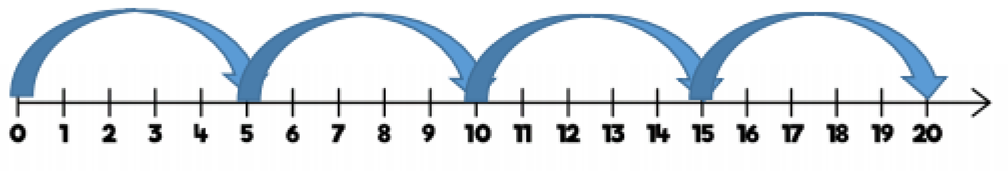
$$18 \div 3 = 6$$

Key points

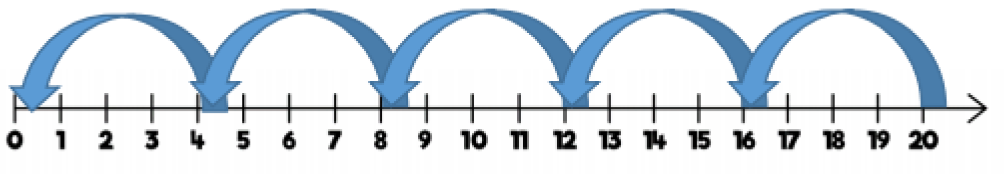
- Support children's counting in multiples, forwards and backwards.
- Children can keep track of their counting by moving counters or cubes along the track.
- Translucent counters can be used to help children see the number they have landed on.
- Children place their counter on 0 to start and then count on to find the product, when multiplying.
- When dividing, children place their counter of the number they are diving and then count back in jumps of the number they are dividing by until they reach 0.
- The number of jumps made provides children with the answer.
- Become less efficient with larger numbers.

Number lines (labelled)





$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

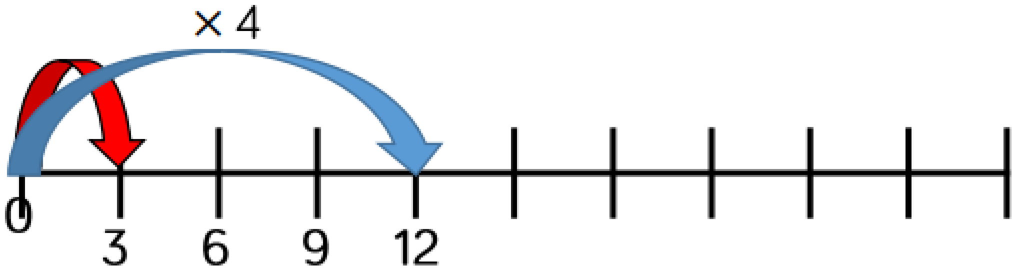


$$20 \div 4 = 5$$

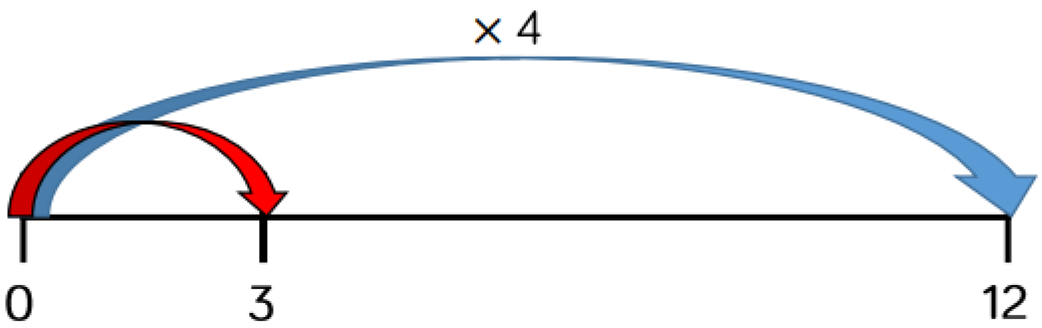
Key points

- Can be used for counting in multiples, forwards and backwards as well as calculating single-digit multiplications.
- Children start at 0 and count on to find the product of numbers when multiplying.
- When dividing, children start at the number they are diving and count back in jumps of the umber they are dividing by until they reach 0.
- The number of jumps made provides children with the answer to the division.
- Become inefficient as numbers become larger.

Number lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?

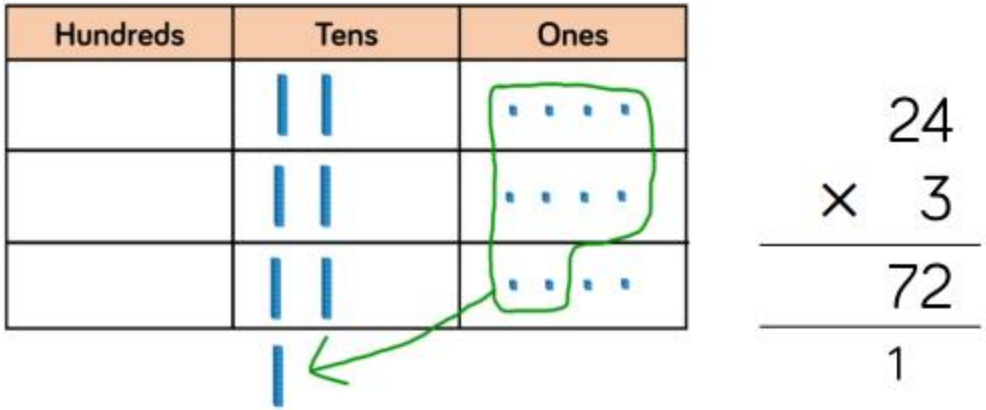


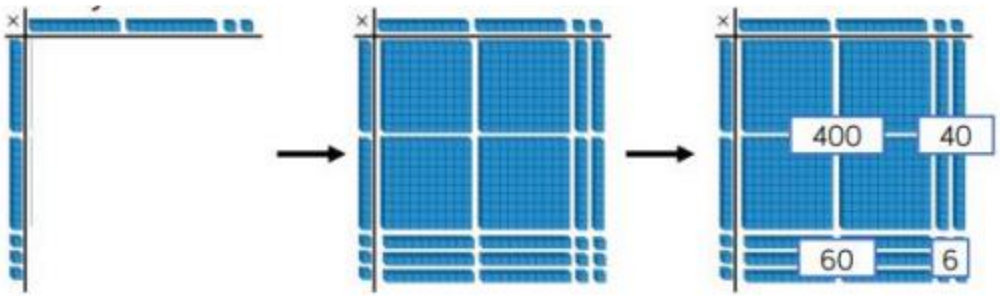
A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Key points

- Can be used to represent scaling as multiplication or division.
- Black number lines with intervals can be used to support children when representing scaling accurately.
- Children can calculate scaling problems by labelling intervals with multiples.
- Children can also use blank number lines without intervals to represent scaling.

Base 10/Dienes (multiplication)

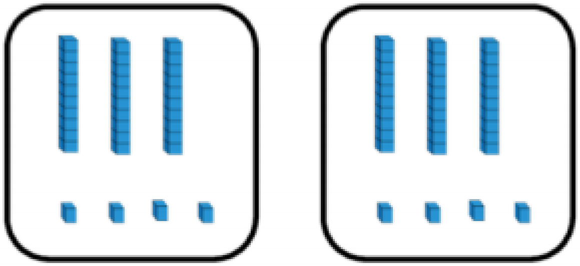




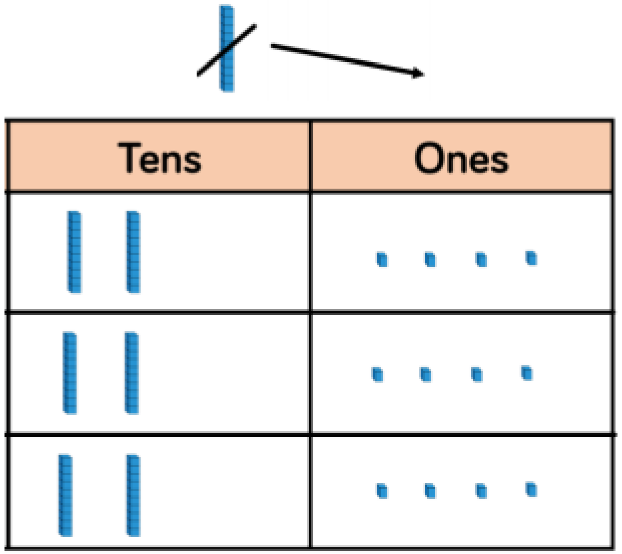
Key points

- Provides an effective way to support children's understanding of column multiplication.
- Children need to write out the formal method when working with concrete resources or pictorial representations to help build their conceptual understanding.
- Become less efficient as the numbers or amounts of groups become larger.
- This is due to the amount of equipment needed and the number of exchanges required.
- Provides support for the area model of multiplication.
- Children build the number in a rectangular shape which they then find the area of by calculating the total value of all the pieces.
- The area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

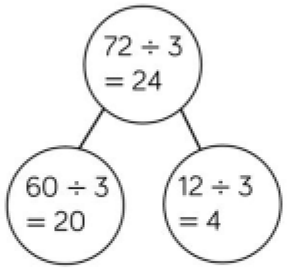
Base 10/Dienes (division)



$68 \div 2 = 34$



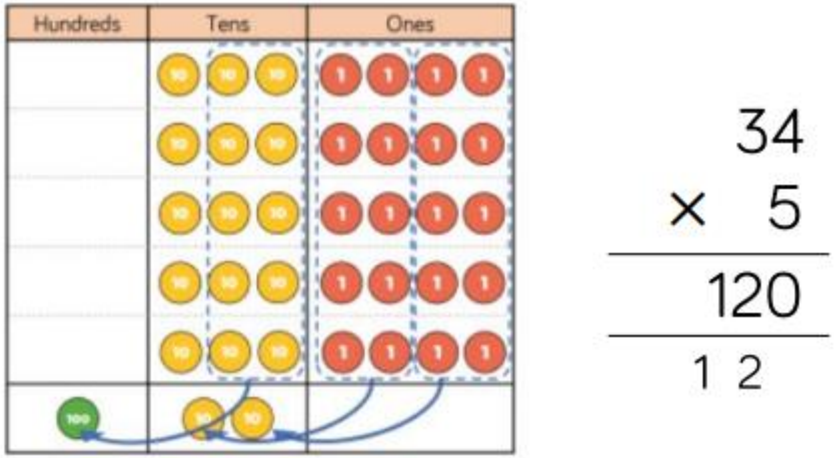
$72 \div 3 = 24$

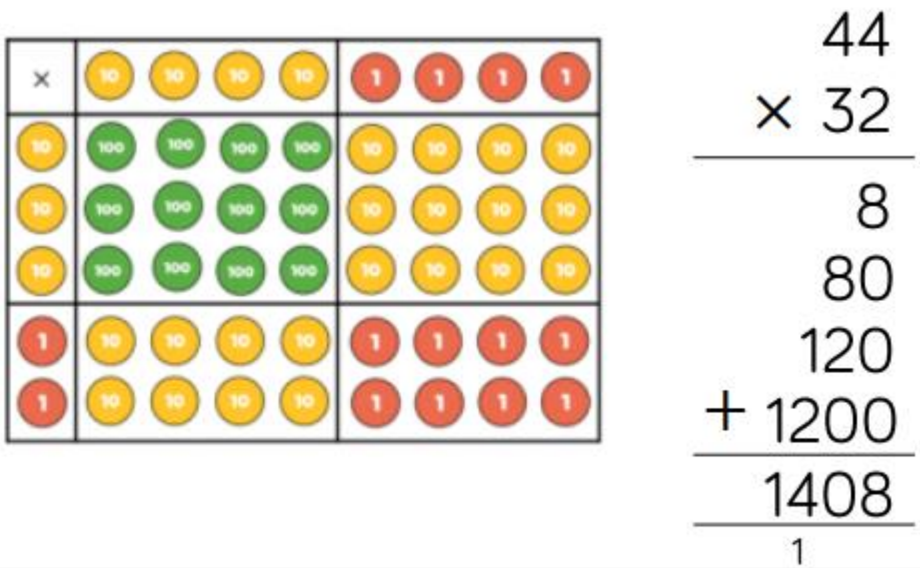


Key points

- As numbers become larger, Base 10/Dienes can be an effective way of moving children on from representing numbers as ones to representing them as tens and ones in order to divide.
- Children can share the equipment between different groups by drawing circles or creating rows on a place value grid.
- When sharing, Children start with the larger place value and work from left to right.
- If there are any left in a column, they exchange.
- For example, one ten for ten ones.
- Encourage children to use the part-whole model when recording so they consider how the number has been partitioned in order to divide.
- This will support them with mental methods.

Place value counters (multiplication)

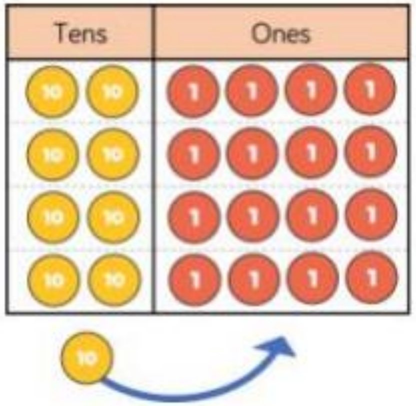


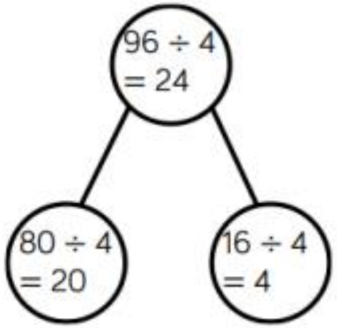


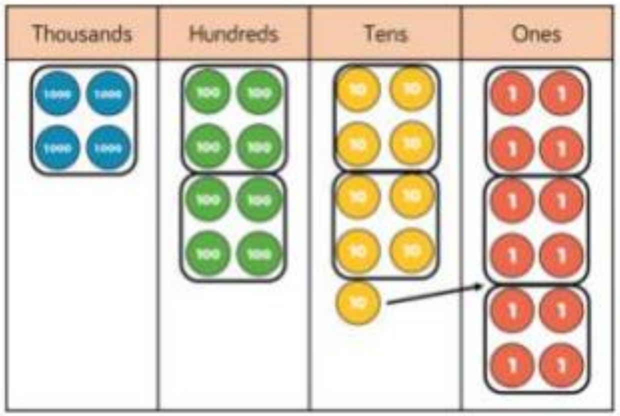
Key points

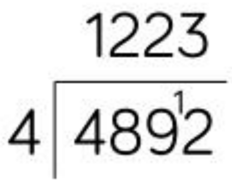
- Provides an effective way to support column multiplication.
- Children will need to write out the formal calculation alongside their resources/pictures to develop their conceptual understanding.
- Counters should replace Base 10/Dienes once the numbers or amount of groups become larger.
- Counters should be used to support the understanding of the written method instead of supporting the arithmetic.
- Place value counters also support the area model of multiplication.
- Children can see how to multiply 2-digit numbers by 2- digit numbers.

Place value counters (division)









Key points

- Children can share place value counters between groups when working with smaller numbers.
- They start by sharing the larger place value column and work from left to right.
- Any counters left over once they have been shared are exchanged.
- Using place value counters also supports children's understanding of short division by grouping the counters instead of sharing them.
- Children work from left to right through the place value columns and group the counters in the number they are dividing by.
- Any counters left over once they have been grouped are exchanged.

Glossary of Terms

Vocabulary	**Definition**
Area	A measure of the size of any plane surface. Area is usually measured in square units. E.g. square centimetres cm2 or square metres m2.
Addend	A number to be added to another.
Array	An ordered collection of counters, number etc. in rows and columns.
Capacity	The volume of a material (typically liquid or air) held within a container.
Common Factor	A number which is a factor of two or more numbers.
Common Fraction	Where the numerator and the denominator are both whole integers.
Composite Shape	A Shape formed by combining two or more shapes.
Concrete objects	Objects that can be handled and manipulated to support mathematical understanding.
Conjecture	An educated guess,
Consecutive	Following in order,
Curved surface.	The curved boundary of a 3-D solid, e.g. the curved surface of a cylinder between to the two circular ends.
Decomposition	A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. In this method the number to be subtracted from (the minuend) is repartitioned, if necessary, in order that each digit of the number to be subtracted (the subtrahend) is smaller than its corresponding digit in the minuend.
Degree	The most common unit of measurement for angle. One whole turn is equal to 360 degrees.
Degree of Accuracy	A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures.

Denomination	The face value of coins. In the smallest denomination of UK currency (known as Sterling) is 1p and the largest denomination of currency is a £50 note.
Difference	In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another.
Directed number	A number having a direction as well as a size e.g. -7, +10, etc. Such numbers can be usefully represented on a number line extending in both directions from zero.
Dividend	In division, the number that is divided. E.g. in 15 ÷ 3, 15 is the dividend.
Divisor	The number by which another is divided. Example: In the calculation 30 ÷ 6 = 5, the divisor is 6. In this example, 30 is the dividend and 5 is the quotient.
Edge	A line segment, joining two vertices of a figure. A line segment formed by the intersection of two plane surfaces. Examples: a square has four edges; and a cuboid has twelve edges.
Efficient Methods	A means of calculation (which can be mental or written) that achieves a correct answer with as few steps as possible. In written calculations this often involves setting out calculations in a columnar layout. If a calculator is used the most efficient method uses as few key entries as possible.
Equivalent Expression	A numerical or algebraic expression which is the same as the original expression but is in a different form which might be more useful as a starting point to solve a particular problem. Example: 6 + 10x is equivalent to 2(3 + 5x); 19 × 21 is equivalent to (20 – 1)(20 + 1) which is equivalent to 202 – 1 which equals 399. Equivalent expressions are identically equal to each other. Often a 3-way equals sign is used to denote ‘is identically equal to’.
Equivalent Fraction	Fractions with the same value as another. For example: 4/8, 5/10, 8/16 are all equivalent fractions and all are equal to ½.

Estimate	1. Verb: To arrive at a rough or approximate answer by calculating with suitable approximations for terms or, in measurement, by using previous experience. 2. Noun: A rough or approximate answer.
Even Number	An integer that is divisible by 2.
Exchange	Change a number or expression for another of equal value. The process of exchange is used in some standard compact methods of calculation. Examples: ‘carrying figures’ in addition, multiplication or division; and ‘decomposition’ in subtraction.
Expression	A mathematical form expressed symbolically. Examples: 7 + 3; a2 + b 2.
Face	One of the flat surfaces of a solid shape. Example: a cube has six faces; each face being a square,
Factorise	To express a number or a polynomial as the product of its factors. Examples: Factorising 12: 12 = 1 × 12 = 2 × 6 = 3 × 4 The factors of 12 are 1, 2, 3, 4, 6 and 12. 12 may be expressed as a product of its prime factors: 12 = 2 × 2 × 3.
Fluency	To be mathematically fluent one must have a mix of conceptual understanding, procedural fluency and knowledge of facts to enable you to tackle problems appropriate to your stage of development confidently, accurately and efficiently.
Graph	A diagram showing a relationship between variables. Adjective: graphical.
Improper Fraction	An improper fraction has a numerator that is greater than its denominator. Example: 9 /4 is improper and could be expressed as the mixed number 2¼.
Inequality	When one number, or quantity, is not equal to another. Statements such as a ≠ b, a b or a≥b are inequalities. The inequality signs in use are: ≠ means ‘not equal to’; A ≠ B means ‘A is not equal to B” < means ‘less than’; A < B means ‘A is less than B’ > means ‘greater than’; A > B means ‘A is greater than B’ ≤ means ‘less than or equal to’; A ≤ B means ‘A is less than

	or equal to B' ≥ means 'greater than or equal to'; A ≥ B means 'A is greater than or equal to B'.
Infinite	Of a number, always bigger than any (finite) number that can be thought of. Of a sequence or set, going on forever. The set of integers is an infinite set.
Interpret	Draw out the key mathematical features of a graph, or a chain of reasoning, or a mathematical model, or the solutions of an equation, etc.
Inverse Operations	Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. 5 + 6 – 6 = 5. Multiplication and division are inverse operations e.g. 6 × 10 ÷ 10 = 6.
Kilo-	Prefix denoting one thousand.
Mass	A characteristic of a body, relating to the amount of matter within it. Mass differs from weight, the force with which a body is attracted towards the earth's centre. Whereas, under certain conditions, a body can become weightless, mass is constant. In a constant gravitational field weight is proportional to mass.
Mean	Often used synonymously with average. The mean (sometimes referred to as the arithmetic mean) of a set of discrete data is the sum of quantities divided by the number of quantities. Example: The arithmetic mean of 5, 6, 14, 15 and 45 is (5 + 6 + 14 + 15 + 45) ÷ 5 i.e. 17.
Mile	An imperial measure of length. 1 mile = 1760 yards. 5 miles is approximately 8 kilometres.
Milli-	Prefix. One-thousandth.
Mixed Fraction	A whole number and a fractional part expressed as a common fraction. Example: $1\frac{1}{3}$ is a mixed fraction. Also known as a mixed number.
Multiple	For any integers a and b, a is a multiple of b if a third integer c exists so that a = bc.

Multiplicand	A number to be multiplied by another. e.g. in 5 × 3, 5 is the multiplicand as it is the number to be multiplied by 3.
Negative Integer	An integer less than 0. Examples: -1, -2, -3 etc.
Notation	A convention for recording mathematical ideas. Examples: Money is recorded using decimal notation e.g. £2.50.
Number Bond	A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.
Number Sentence	A mathematical sentence involving numbers. Examples: 3 + 6 = 9.
Numeral	A symbol used to denote a number. The Roman numerals I, V, X, L, C, D and M represent the numbers one, five, ten, fifty, one hundred, five hundred and one thousand. The Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used in the Hindu-Arabic system giving numbers in the form that is widely used today.
Numerator	In the notation of common fractions, the number written on the top – the dividend (the part that is divided). In the fraction ⅔, the numerator is 2.
Odd Number	An integer that has a remainder of 1 when divided by 2.
Order of Operation	This refers to the order in which different mathematical operations are applied in a calculation. This convention is often encapsulated in the mnemonic BODMAS or BIDMAS: Brackets Orders / Indices (powers) Division & Multiplication Addition & Subtraction.
Ordinal Number	A term that describes a position within an ordered set. Example: first, second, third, fourth … twentieth etc.
Parallel	In Euclidean geometry, always equidistant. Parallel lines, curves and planes never meet however far they are produced or extended.
Partition	1. To separate a set into subsets. 2. To split a number into component parts. Example: the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19. 3. A model of division. Example: 21 ÷ 7 is treated as ‘how many sevens in 21?’

Pattern	A systematic arrangement of numbers, shapes or other elements according to a rule.
Place Value	The value of a digit that relates to its position or place in a number. Example: in 1482 the digits represent 1 thousand, 4 hundreds, 8 tens and 2 ones respectively; in 12.34 the digits represent 1 ten, 2 ones, 3 tenths and 4 hundredths respectively.
Polygon	A closed plane figure bounded by straight lines. The name derives from many angles. If all interior angles are less than 180° the polygon is convex. If any interior angle is greater than 180°, the polygon is concave. If the sides are all of equal length and the angles are all of equal size, then the polygon is regular; otherwise it is irregular. Adjective: polygonal.
Power of Ten	1. 100 (i.e. 102 or 10 x 10) is the second power of 10, 1000 (i.e.103 or 10 x 10 x 10) is the third power of 10 etc. Powers of other numbers are defined in the same way. Example: 2 (21), 4 (22), 8 (23), 16 (24) etc are powers of 2.
Prime Factor	The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 (12 = 2 x 2 x 3).
Prime Number	A whole number greater than 1 that has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).
Product	The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since 2 × 3 = 6.
Quotient	The result of a division. Example: 46 ÷ 3 = 15⅓ and 15⅓ is the quotient of 46 by 3. Where the operation of division is applied to the set of integers, and the result expressed in integers, for example 46 ÷ 3 = 15 remainder 1 then 15 is the quotient of 46 by 3 and 1 is the remainder.
Ratio	A part-to-part comparison. The ratio of a to b is usually written a : b. Example: In a recipe for pastry fat and flour are mixed in the ratio 1 : 2 which means that the fat used has half the mass of the flour, that is amount of fat/amount of flour = ½. Thus ratios are equivalent to particular fractional parts.

Remainder	In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1.
Repeated Addition	The process of repeatedly adding the same number or amount. One model for multiplication. Example 5 + 5 + 5 + 5 = 5 x 4.
Repeated Subtraction	The process of repeatedly subtracting the same number or amount. One model for division. Example 35 -5 - 5 - 5 - 5 - 5 - 5 - 5 = 0 so 35 ÷ 5 = 7 remainder 0.
Sequence	A succession of terms formed according to a rule. There is a definite relation between one term and the next and between each term and its position in the sequence. Example: 1, 4, 9, 16, 25 etc.
Share (equally)	One model for the process of division.
Sort	To classify a set of entities into specified categories.
Square Number	A number that can be expressed as the product of two equal numbers. Example 36 = 6 x 6 and so 36 is a square number or “6 squared”. A square number can be represented by dots in a square array.
Subtrahend	A number to be subtracted from another.
Volume	A measure of three-dimensional space. Usually measured in cubic units; for example, cubic centimetres (cm3) and cubic metres (m3).